



*a New Day for Federal Service*

# Multivariate Tests for Phase Capacity

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<sup>1</sup>The opinions, findings, and conclusions expressed in this presentation are those of the author and do not necessarily reflect those of the U.S. Office of Personnel Management.

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# Outline

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# I. Background

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# Nonresponse and Nonrespondent Follow-Up

- Invariably, not all sampled units respond to the initial survey solicitation
- Most surveys repeatedly follow-up with nonrespondents making additional mailings, phone calls, household visits, etc., often chasing a preset response rate target
- Each subsequent reminder brings in a new “wave” of data, which tends to be progressively smaller in size, thereby impacting estimates less and less
- Other temporal delineations of waves possible

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# The Notion of Phase Capacity

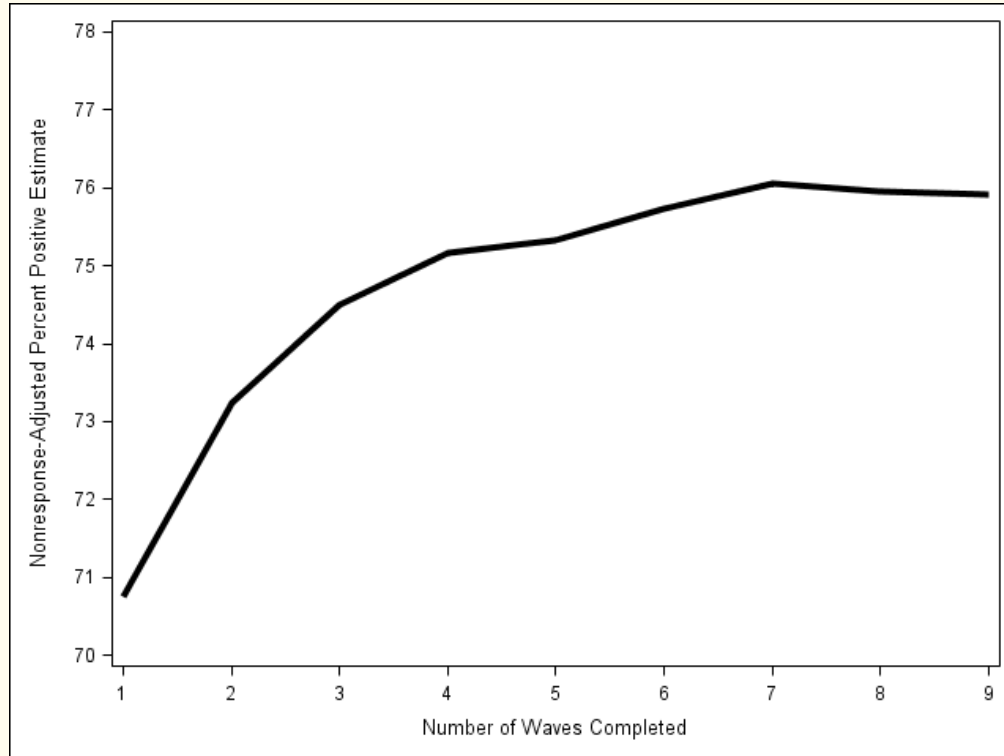
- In their discussion of responsive survey design, Groves and Heeringa (2006) define the following key terms:
  - *design phase* – spell of data collection period with stable frame, sample, and recruitment protocol
  - *phase capacity* – point during a design phase at which additional responses cease influencing key statistics
- Rather than fixating on a target response rate, they argue one should change design phases (e.g., switch mode, increase incentive) or discontinue nonrespondent follow-up altogether once phase capacity has been reached
- Problem for practitioners: no calculable rule given

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# Illustration of Phase Capacity in the Federal Employee Viewpoint Survey (FEVS)

- The FEVS is an annual organizational climate survey administered by the U.S. Office of Personnel Management (OPM) to a sample of 800,000+ federal employees from 80+ agencies
- Web-based instrument comprised mainly of attitudinal items posed on a five-point Likert scale
- Key statistics are “percent positive” estimates based on the dichotomization of, for example, “Completely Agree” or “Agree” elections versus all other possible response choices
- Nonrespondents are sent weekly reminder emails

# Example of a Nonresponse-Adjusted Percent Positive Trend Using Cumulative Responses



← Goal is to identify point estimate stability at earliest possible wave

Note: estimate stability does not necessarily imply that the value converged upon is free of nonresponse error; it implies that additional follow-ups under the same protocol will continue to be inefficacious

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## II. Brief Summary of Prior Research – Univariate Phase Capacity Tests



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# Previously Proposed Univariate Tests

- Rao, Glickman, and Glynn (RGG) (2008) (termed “stopping rules”) – best-performing method used multiple imputation (MI)
- Idea is to multiply impute (Rubin, 1987) the missing data  $M$  ( $M \geq 2$ ) times for nonrespondents as of wave  $k$ , then delete responses obtained during wave  $k$ , specifically, and repeat for nonrespondents as wave  $k - 1 \rightarrow$  result is  $2M$  completed data sets and two nonresponse-adjusted, MI point estimates
- A  $t$ -test is carried out by dividing the two point estimates’ difference by an estimate of the MI variance of the difference – see Appendix A of Lewis (2014a) for example
- Phase capacity declared once the test statistic is insignificant

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## Previously Proposed Univariate Tests (2)

- RGG approach is limited in that it is only designed to track a sample mean and inapplicable to surveys that conduct weighting adjustments for nonresponse
- Lewis (2014b) describes a new method circumventing these limitations: same premise, except nonresponse-adjusted point estimates are formulated based on two sets of weights, one for respondents through wave  $k$  and another for respondents through wave  $k - 1$
- As with the RGG approach, tricky part is deriving a variance factoring in the covariance attributable to shared respondent set through wave  $k - 1$
- Three viable methods to do so are discussed: (1) Taylor series linearization; (2) simple linear regression on a stacked data set; and (3) replication



# III. Multivariate Extensions of Phase Capacity Tests

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# Background

- A practical limitation of both the RGG approach and Lewis' variant is that they are univariate in nature → how would one proceed if independently conducted on two or more point estimates with conflicting results?
- Chapter 4 of Lewis (2014a) proposes two multivariate methods to provide a single yes/no answer for a battery of  $D$  point estimates:
  1. Wald Chi-Square Method – direct multivariate extension of two-sample  $t$ -test using matrix algebra
  2. Non-Zero Trajectory Method – based on ideas of longitudinal data analysis (Singer and Willett, 2003), jointly fit  $D$  simple linear regression models of point estimates' relative percent change
- Both methods default to treating each point estimate difference equivalently, but differential importance can be assigned to each via a contrast vector

# Wald Chi-Square Method

- Let  $\mathbf{D}$  denote a  $D \times 1$  matrix of nonresponse-adjusted point estimate differences, and let  $\mathbf{S}$  denote the corresponding  $D \times D$  variance-covariance matrix
- Entries of  $\mathbf{S}$  can be obtained via Taylor series linearization or replication (i.e., as discussed in Lewis (2014b))
- Supposing the goal is to test for no significant differences, the test statistic is

$$\chi_W^2 = \mathbf{D}^T \mathbf{S}^{-1} \mathbf{D}$$

which is referenced against a chi-square distribution with  $D - 1$  degrees of freedom

# Non-Zero Trajectory Method

- Find the  $D$  differences' 3 most recent relative percent changes (to harmonize potential scale incongruities):

Wave	Item 4	Item 4 Rel % Chg	Item 5	Item 5 Rel % Chg	Item 13	Item 13 Rel % Chg
$k - 3$	75.2%	--	83.6%	--	88.5%	--
$k - 2$	75.3%	0.2%	83.8%	0.2%	88.6%	0.1%
$k - 1$	75.7%	0.5%	83.9%	0.2%	88.6%	0.0%
$k$	76.1%	0.4%	84.2%	0.3%	88.7%	0.2%

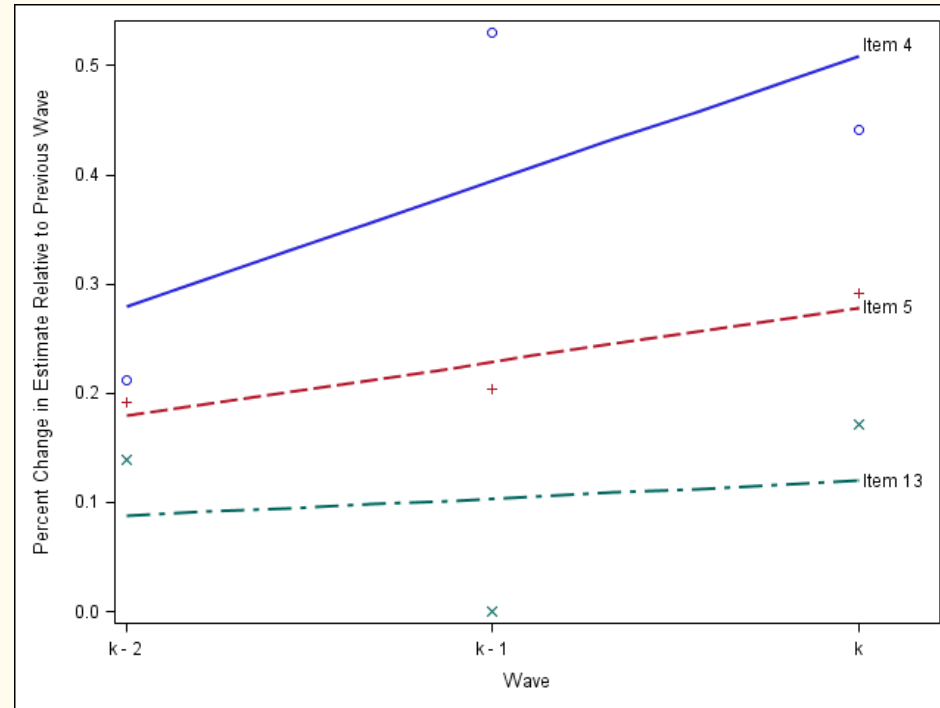
- Treating  $w$  as a wave indicator one unit apart (e.g., 1, 2, 3), one then estimates the following model:

$$\Delta_d = \beta_{01} + \beta_{02} + \dots + \beta_{0D} + \beta_{11}w + \beta_{12}w + \dots + \beta_{1D}w + \varepsilon_d$$

where the first set of  $D$  terms represent estimate-specific intercepts, and the second set represents estimate-specific slopes

- Disadvantage: at least 4 waves needed (Wald needs 2)


# Visualization of Non-Zero Trajectory Method



- If point estimates have stabilized, we would expect all model coefficients to be insignificantly different from zero; we can test for this using a traditional linear model  $F$  test

$$F = \hat{\beta}^T (\text{cov}(\hat{\beta}))^{-1} \hat{\beta}$$

which can be referenced against an  $F$  distribution with  $D$  and  $2D$  degrees of freedom, respectively

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# IV. Retrospective Application using the 2011 Federal Employee Viewpoint Survey



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# FEVS 2011 Application Details

- Batteries of point estimates investigated were the four Human Capital Assessment and Accountability Framework (HCAAF) indices, which are averages of the percent positive estimates of thematically-linked items (e.g., Job Satisfaction, Talent Management)
- Using timestamp information for three agencies, respondents were apportioned into waves, and each successive (accumulating) set of respondents was assigned a set of weights raked to known marginal distributions from sample frame (e.g., agency component, minority status, gender, and supervisory status)
- Retroactively implemented the two methods for each agency x index combination to compare and contrast performance

# FEVS 2011 Application Results

Index	<i>Method: Wald Chi-Square</i>			<i>Method: Non-Zero Trajectory</i>		
	Stopping Wave	Estimate	NR Error	Stopping Wave	Estimate	NR Error
<i>Agency 1</i>						
JS	4	68.5	-0.6	6	68.8	-0.2
LKM	3	60.2	-1.4	9	61.6	0.0
ROPC	2	53.6	-2.6	9	56.2	0.0
TM	5	59.9	-0.7	9	60.6	0.0
<i>Agency 2</i>						
JS	2	69.8	-1.0	5	71.0	0.1
LKM	2	72.8	-0.4	5	73.1	0.1
ROPC	4	66.3	0.1	5	66.4	0.2
TM	2	68.7	-1.3	5	70.0	0.1
<i>Agency 3</i>						
JS	3	73.1	-0.7	6	73.5	-0.3
LKM	2	70.5	-1.3	7	71.5	-0.2
ROPC	4	63.7	-0.6	5	63.8	-0.5
TM	2	69.4	-1.0	6	70.2	-0.2

- Wald method concludes phase capacity earlier, in part because it requires fewer waves (2 vs. 4 for NZT); this results in larger residual differences relative to the final wave estimate (see NR Error column) – recall there is an upward trend in the point estimates underlying indices


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# V. Limitations and Further Research

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# Practical Limitations

- Actual adoption of these approaches in FEVS would face resistance because:
  - Desirable to treat each agency equitably; beginning in FEVS 2012, field period was preset to 6 weeks for all agencies
  - Higher scores are better, and so there may be opposition to any change, shortened field period included, believed to reduce point estimates
- Data must be collected/processed real-time, and it was tacitly assumed that the full sample is “active” – may be impractical for in-person surveys covering a vast geographical expanse taking weeks or months for interviewers to exhaust sample cases, although tests could be applied to subsamples

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## Practical Limitations (2)

- Even when entire sample is “active,” may not be feasible to send reminders simultaneously as in the FEVS Web mode – alternative data collection wave definition may be a plausible work-around
- Despite aversion to phrase *stopping rule*, stopping was the only design phase change investigated in this research – would be interesting to apply in a sequential mixed-mode survey setting or in surveys with two stages of data collection, such as the National Immunization Survey (NIS) or the Residential Energy Consumption Survey (RECS)
- In both of those surveys, the preeminent estimates are those derived from secondary data collection stage, medical records (NIS) and energy suppliers (RECS); hence, one might want the tests to have differential sensitivities

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# Further Research

- A general limitation of the two traditional perspectives of nonresponse (deterministic vs. stochastic) is that the act of responding is considered a dichotomous event
- Chapter 2 of Lewis (2014a) extends the familiar sample mean nonresponse error/bias theory to account for a time dimension:
  - *Deterministic perspective* – conceptualize sample frame as composed of  $K + 1$  mutually exclusive domains, units that always respond during wave  $k$  ( $k = 1, \dots, K$ ), specifically, and a domain for units that never respond
  - *Stochastic perspective* – partition a unit's traditional response propensity into a vector of  $K$  wave-specific propensities, the sum of which constitutes its overall propensity
- To be presented at the TSE15 conference later this year

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## Further Research (2)

- Wagner and Raghunathan (2010) proposed a prospective stopping rule, aiming to quantify the likelihood a pending wave of follow-up will change a point estimate more than some predetermined amount
- Chapter 5 of Lewis (2014a) points out several limitations and introduces a more general approach; unfortunately, results were lackluster in simulation and application, even when the expected value of the point estimate was stable over the data collection period
- Applications of time series analysis and forecasting could prove fruitful, especially if predictions beyond wave  $k + 1$  are desired

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# Thanks!

Questions/Comments?  
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