

# Multiple Comparisons: To Adjust or Not to Adjust?

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# Comparing State Means

- Data users often wish to compare American Community Survey (ACS) estimates among states
- The Census Bureau publishes ranking tables that compare states using statistical hypothesis tests
- Such statistical hypothesis tests may produce excess false significant differences

- Let  $\mu_X$  be the average commute time to work in state X
- Let  $\hat{\mu}_X$  be the ACS estimate of  $\mu_X$
- The null ( $H_0$ ) and alternative ( $H_A$ ) hypotheses for comparing states X and Y are:

$$H_0 : \mu_X = \mu_Y$$

$$H_A : \mu_X \neq \mu_Y$$

- This leads to the test statistic  $Z_{test}$ :

$$Z_{test} = \frac{(\hat{\mu}_X - \hat{\mu}_Y)}{SE(\hat{\mu}_X - \hat{\mu}_Y)} \quad (1)$$

- Comparing the average commute time in Virginia to each of the other states and D.C. requires 50 separate tests
- Each of these tests has a false positive rate capped at 10%
- Despite this, we may end up with a higher false positive rate in *the group of tests*
- If all states had the same commute times, the probability that at least one pair of states will falsely appear to be different is  $(1 - (1 - 0.1)^{50}) \times 100 \approx 99.5\%$ !

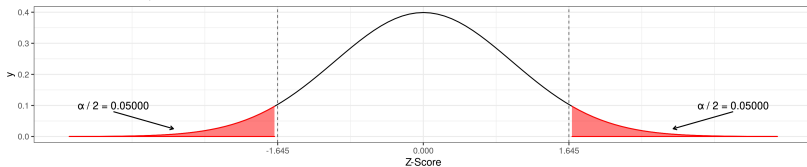
# Z Thresholds for Rejecting $H_0$

- One way to prevent this is to require a larger difference between estimates to conclude that two states are different
- A larger difference between estimates corresponds to a larger absolute value of  $Z_{test}$
- There are several ways to adjust our threshold so that the false positive rate of a group of tests is capped at 10%
- We consider three thresholds for rejection:
  - No Adjustment (Reject  $H_0$  if  $|Z_{test}| > 1.645$ )
  - Independence (Reject  $H_0$  if  $|Z_{test}| > 3.075$ )
  - Bonferroni (Reject  $H_0$  if  $|Z_{test}| > 3.090$ )

# Rejection Regions by Adjustment

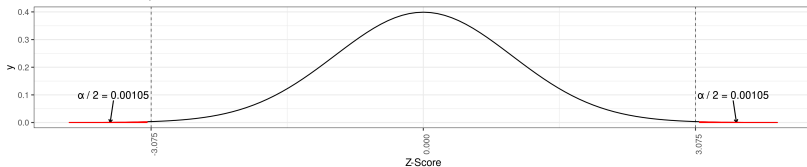
Rejection Region (No Adjustment)

Critical Z-Score:  $\pm 1.645$ ;  $\alpha = 0.1000$



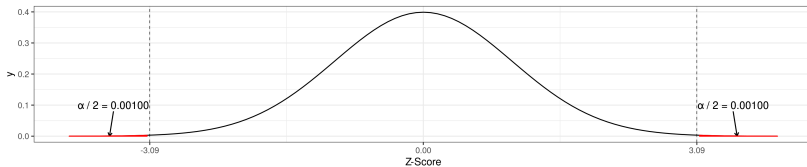
Rejection Region (Independence Adjustment)

Critical Z-Score:  $\pm 3.075$ ;  $\alpha = 0.0021$



Rejection Region (Bonferroni Adjustment)

Critical Z-Score:  $\pm 3.090$ ;  $\alpha = 0.0020$



# Comparison Differences

Figure: Time to Work (R0801): Virginia vs. Other States

(a) No Adjustment

	Estimate	Standard Error
51. New York	34.0	0.1
50. Maryland	33.7	0.3
49. New Jersey	33.1	0.2
48. District of Columbia	31.7	0.5
47. Massachusetts	31.0	0.3
46. California	30.7	0.1
45. Illinois	29.6	0.2
44. Georgia	29.3	0.3
43. Virginia	29.1	0.2
42. Washington	28.8	0.2
41. Florida	28.4	0.2
40. Hawaii	28.1	0.6
39. New Hampshire	28.0	0.6
38. Pennsylvania	27.8	0.2
37. Texas	27.2	0.1
36. Connecticut	27.0	0.4
35. Delaware	26.7	0.7
34. Arizona	26.6	0.2
33. Louisiana	26.6	0.3
32. Colorado	26.5	0.2
31. West Virginia	26.4	0.4
30. South Carolina	26.0	0.3
29. Tennessee	25.9	0.2
28. North Carolina	25.7	0.2
27. Rhode Island	25.7	0.6
26. Nevada	25.6	0.3
25. Alabama	25.4	0.3
24. Michigan	25.1	0.2
23. Mississippi	25.1	0.5
22. Maine	24.6	0.5
21. Minnesota	24.4	0.2
20. Missouri	24.3	0.2
19. Ohio	24.3	0.1
18. Indiana	24.2	0.2
17. Kentucky	24.2	0.3
16. Oregon	24.2	0.3

(b) Independence

	Estimate	Standard Error
51. New York	34.0	0.1
50. Maryland	33.7	0.3
49. New Jersey	33.1	0.2
48. District of Columbia	31.7	0.5
47. Massachusetts	31.0	0.3
46. California	30.7	0.1
45. Illinois	29.6	0.2
44. Georgia	29.3	0.3
43. Virginia	29.1	0.2
42. Washington	28.8	0.2
41. Florida	28.4	0.2
40. Hawaii	28.1	0.6
39. New Hampshire	28.0	0.6
38. Pennsylvania	27.8	0.2
37. Texas	27.2	0.1
36. Connecticut	27.0	0.4
35. Delaware	26.7	0.7
34. Arizona	26.6	0.2
33. Louisiana	26.6	0.3
32. Colorado	26.5	0.2
31. West Virginia	26.4	0.4
30. South Carolina	26.0	0.3
29. Tennessee	25.9	0.2
28. North Carolina	25.7	0.2
27. Rhode Island	25.7	0.6
26. Nevada	25.6	0.3
25. Alabama	25.4	0.3
24. Michigan	25.1	0.2
23. Mississippi	25.1	0.5
22. Maine	24.6	0.5
21. Minnesota	24.4	0.2
20. Missouri	24.3	0.2
19. Ohio	24.3	0.1
18. Indiana	24.2	0.2
17. Kentucky	24.2	0.3
16. Oregon	24.2	0.3

(c) Bonferroni

	Estimate	Standard Error
51. New York	34.0	0.1
50. Maryland	33.7	0.3
49. New Jersey	33.1	0.2
48. District of Columbia	31.7	0.5
47. Massachusetts	31.0	0.3
46. California	30.7	0.1
45. Illinois	29.6	0.2
44. Georgia	29.3	0.3
43. Virginia	29.1	0.2
42. Washington	28.8	0.2
41. Florida	28.4	0.2
40. Hawaii	28.1	0.6
39. New Hampshire	28.0	0.6
38. Pennsylvania	27.8	0.2
37. Texas	27.2	0.1
36. Connecticut	27.0	0.4
35. Delaware	26.7	0.7
34. Arizona	26.6	0.2
33. Louisiana	26.6	0.3
32. Colorado	26.5	0.2
31. West Virginia	26.4	0.4
30. South Carolina	26.0	0.3
29. Tennessee	25.9	0.2
28. North Carolina	25.7	0.2
27. Rhode Island	25.7	0.6
26. Nevada	25.6	0.3
25. Alabama	25.4	0.3
24. Michigan	25.1	0.2
23. Mississippi	25.1	0.5
22. Maine	24.6	0.5
21. Minnesota	24.4	0.2
20. Missouri	24.3	0.2
19. Ohio	24.3	0.1
18. Indiana	24.2	0.2
17. Kentucky	24.2	0.3
16. Oregon	24.2	0.3

$$Z_{noadj}^* = 1.645$$

$$Z_{indep}^* = 3.075$$

$$Z_{bonf}^* = 3.090$$

- The ACS ranking tables may want to investigate the use of adjustments in making these comparisons