

# Imputing seasonal data in an advanced indicator with forecasts from X-13ARIMA-SEATS

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# Monthly Retail Trade



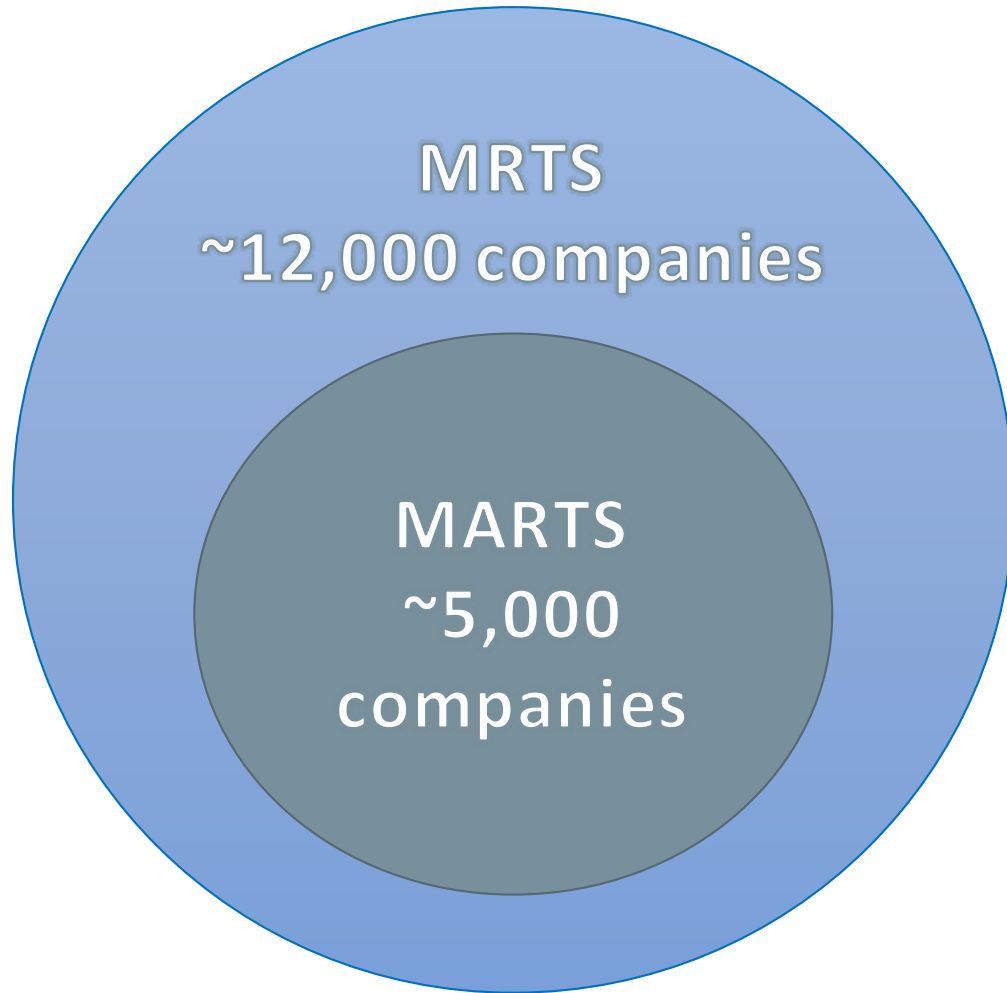
- MARTS - Advance Monthly Retail Trade Survey
  - Economic indicator
  - Input into GDP estimates
  - Collects sales
  - Published approximately 9 business days after the reference month
  - Approximately 7 days to respond
    - Low response rates < 50%

# Monthly Retail Trade



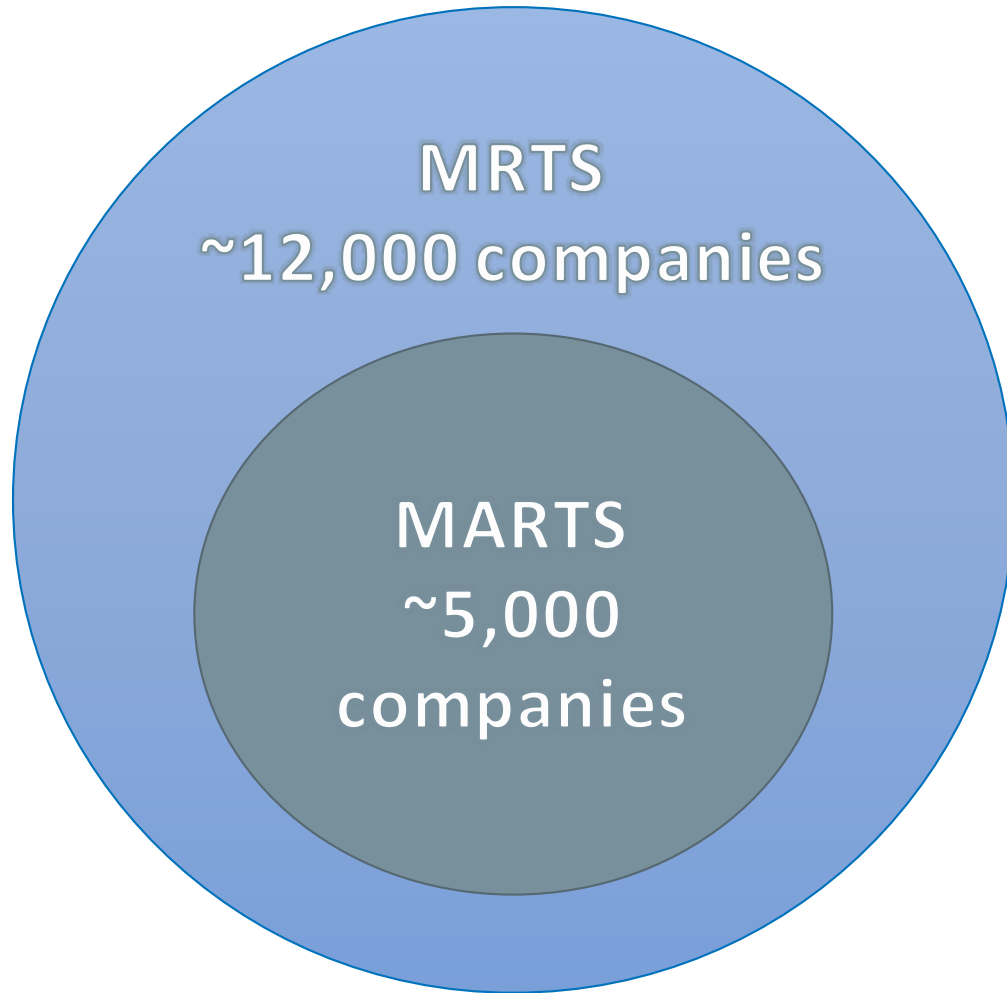
- MARTS - Advance Monthly Retail Trade Survey
  - Selected (large) nonresponding companies imputed by analysts
  - Link relative estimator

# Monthly Retail Trade



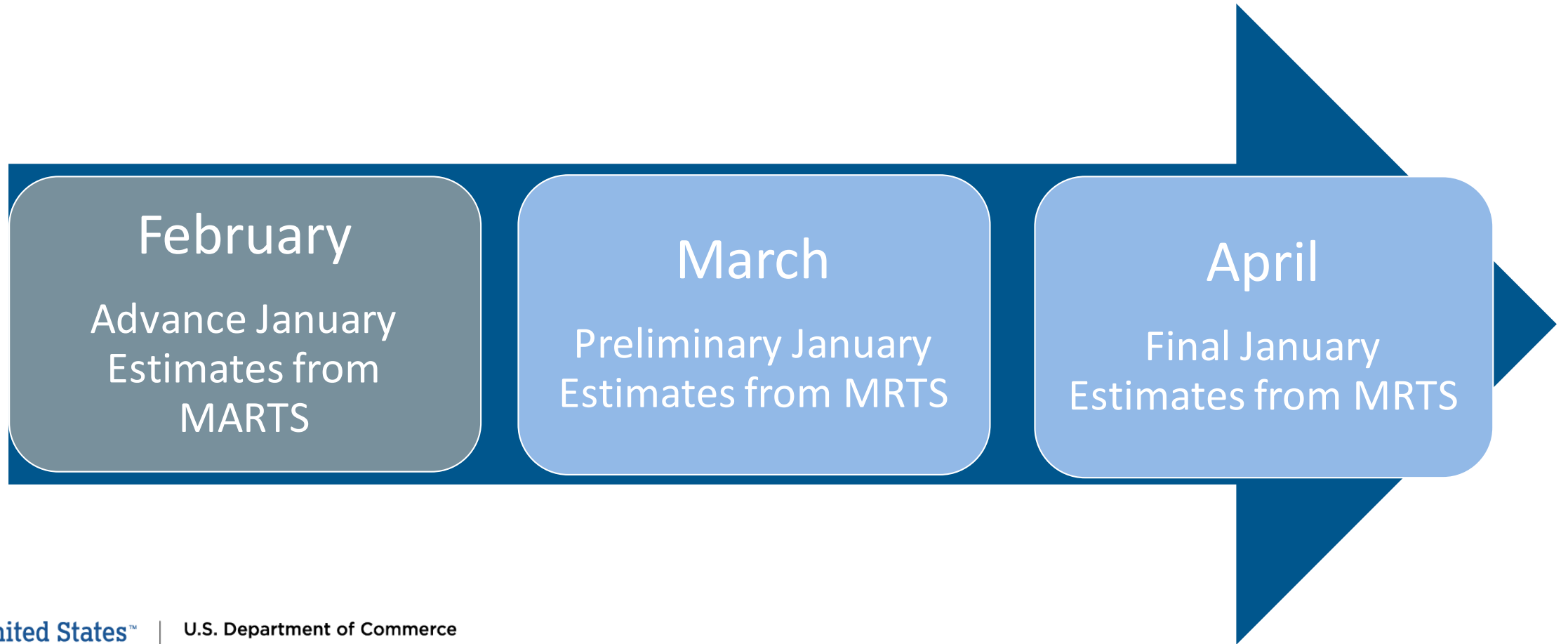
- MRTS – Monthly Retail Trade Survey
  - Input into GDP estimates
  - Collects sales and inventories
  - Published 6 weeks after reference month
  - Approximately 5 weeks to respond

# Monthly Retail Trade



- MRTS – Monthly Retail Trade Survey
  - Analyst imputes retained, ratio imputation for remaining nonrespondents
  - Horvitz-Thompson estimator

# Retail Trade Indicator Release Example for January



# Link Relative Estimator

$$\hat{Y}_{LR,t} = B_{t-1} \left[ \frac{\sum_{i \in C} w_i y_{t,i}}{\sum_{i \in C} w_i y_{t-1,i}} \right]$$

- $\left[ \frac{\sum_{i \in C} w_i y_{t,i}}{\sum_{i \in C} w_i y_{t-1,i}} \right]$  = the link relative ratio
- $\hat{Y}_{LR,t}$  = link relative estimate of the total for period  $t$
- $B_{t-1}$  = benchmark value associated with period  $t - 1$ 
  - MRTS Preliminary total
- $C$  = set of units with usable data for both periods  $t$  and  $t - 1$
- $w_i$  = sample weight for unit  $i$
- Introduced by Madow and Madow (1978)

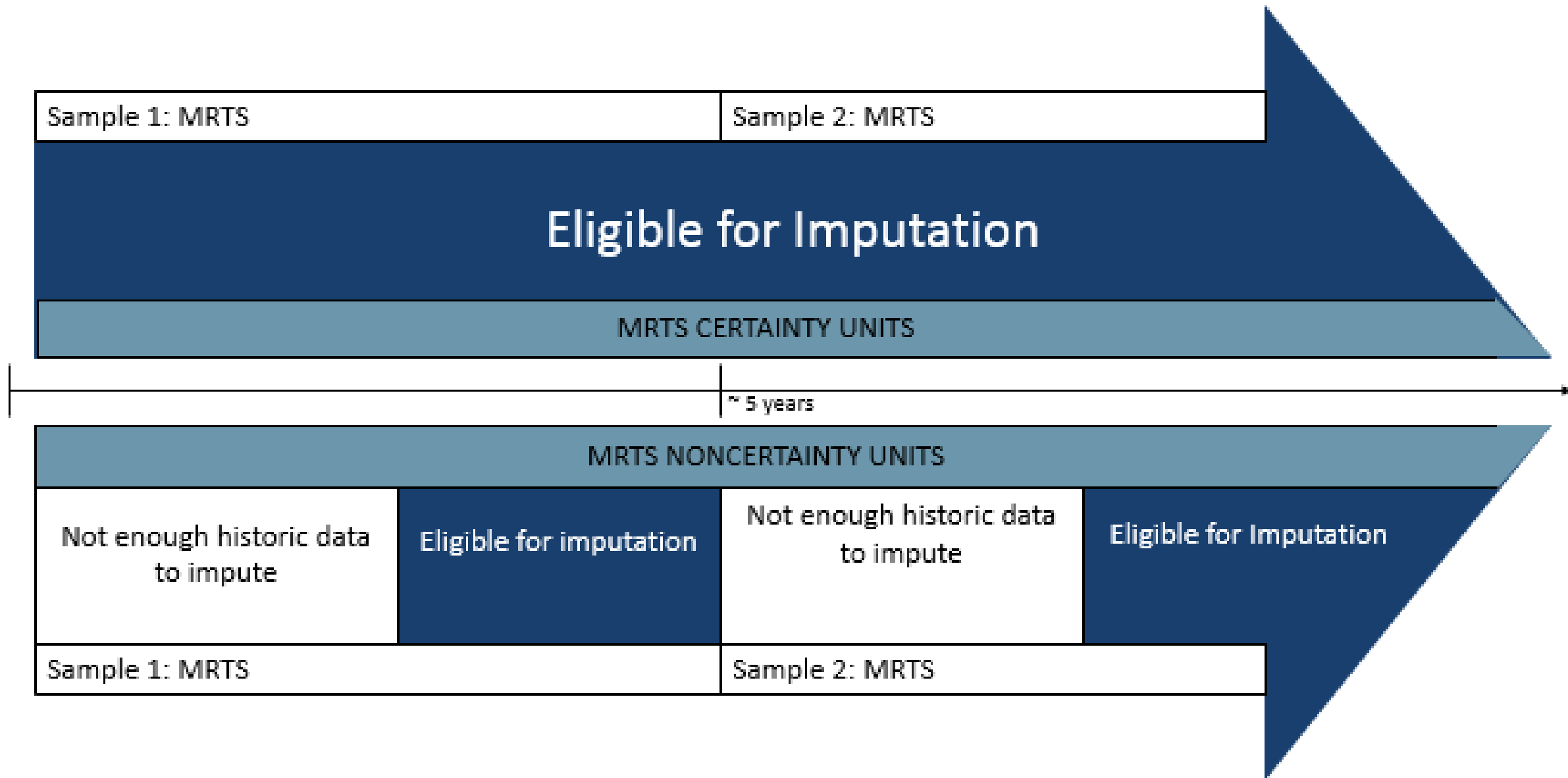
# Developing an imputation method for MARTS

- What are the characteristics of the microdata?
  - Seasonal
  - Trading day effects
  - Moving holiday effects
  - Skewed



# Developing an imputation method for MARTS

- What data are available?
  - Time series of estimated industry totals
  - Unit level monthly historic data from MRTS



# RegARIMA Forecast Approach

- PROC X13
- Fit regARIMA models for each MARTS industry time series
- Extract ARIMA model parameters and calendar adjustment factors
- Compute one-step-ahead unit level forecasts = imputed values
- Quality control on model input and output

# RegARIMA Models

- Regression + ARIMA

$$\ln(Y_t) = \hat{\beta}' X_t + Z_t$$

where  $Y_t$  = time series -> MARTS industry total sales

$X_t$  = regression matrix [trading day, automatic outliers  
moving holiday effects (Easter, Labor Day, Thanksgiving )]

$\hat{\beta}'$  = parameter estimates

$Z_t$  = ARIMA process

[(1 1 0)(1 0 0), uses data at t-1, t-2, t-12, t-13, t-14 ]

# Imputing with a regARIMA model

$$z_{i,t-1} = \ln \left( \frac{y_{i,t-1}}{a_{t-1}} \right) \quad y_{i,t-1} = \text{sales for unit } i \text{ at } t - 1$$

$a_{t-1}$  = calendar adjustment factor at  $t - 1$   
(trading day and moving holiday effects)

$$\tilde{z}_{i,t} = (1 + \varphi_1)z_{i,t-1} - \varphi_1 z_{i,t-2} + \varphi_{12} z_{i,t-12} - (\varphi_1 \varphi_{12} + \varphi_{12}) z_{i,t-13} + \varphi_1 \varphi_{12} z_{i,t-14}$$

$$\tilde{y}_{i,t} = a_t e^{\tilde{z}_{i,t}}$$

# Study design

- January –December 2016
- Calculate observed MARTS response rates for each month
  - 3 subgroups within industry – large certainty, small certainty, noncertainty
- Monthly datasets of MARTS units with reported data
- Induce nonresponse 500x – missing at random using observed response rates as response propensities within industry subgroups
- Impute and calculate link relative ratio with imputed values
  - Impute all units
  - Impute MRTS certainty units only

# Evaluation

- Mean square prediction error (MSPE)
  - Evaluation at the unit level
  - Compare regARIMA forecast performance to implied ratio impute from link relative estimator
  - Computed separately for MRTS certainty and MRTS noncertainty units
- Mean absolute error (MAE) of link relative ratio
  - Evaluation at the tabulation level
  - Compare estimates of link relative ratio to “true” link relative ratio from completed data set

# Results: Three patterns

- No improvement with forecast imputation
- Improvement with forecast imputation for all units
- Improvement with forecast imputation for MRTS certainty units only

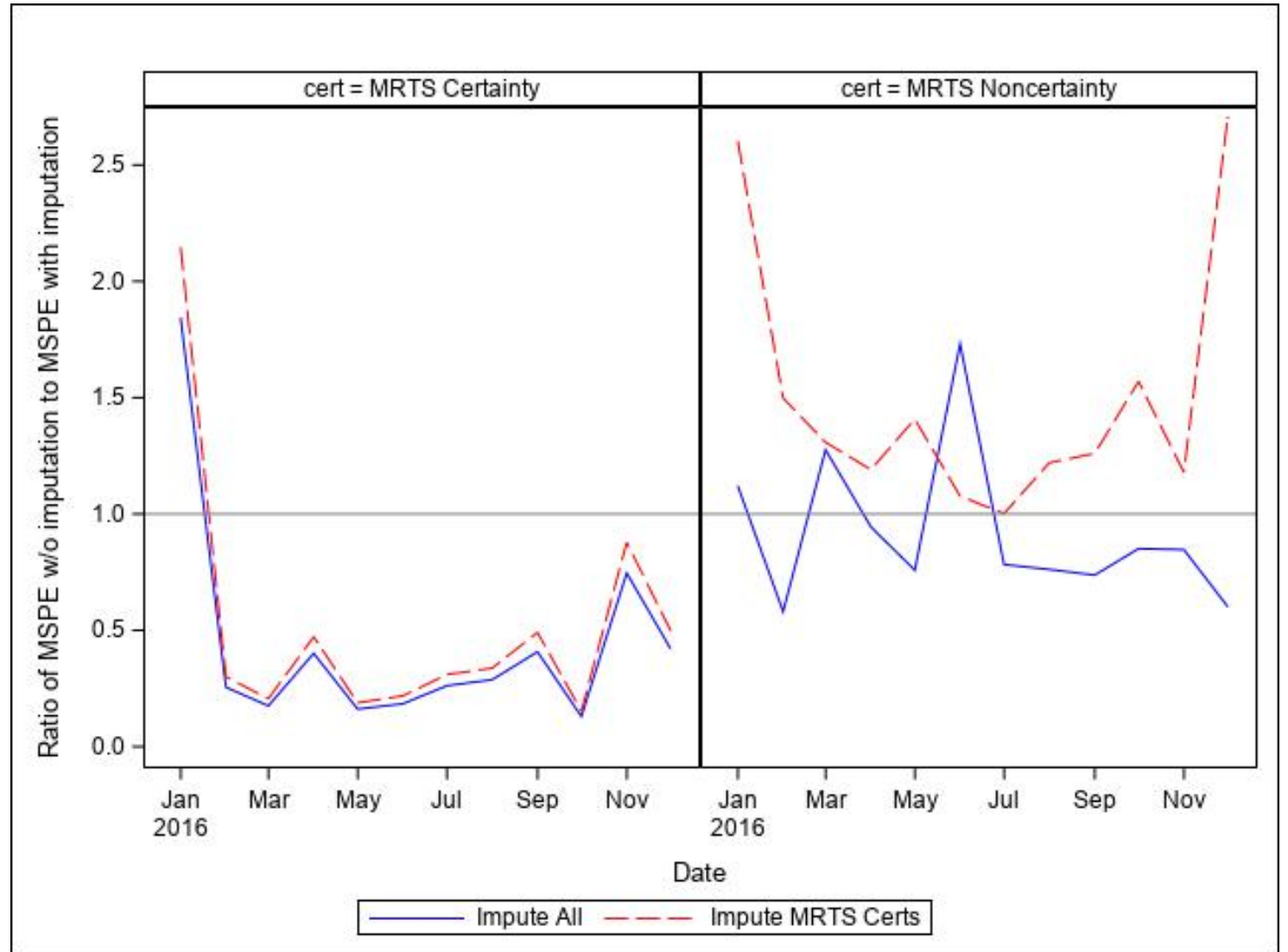


# Unit level prediction errors: No imputation better

$$\frac{MSPE_{noimp}}{MSPE_{impute}}$$

NAICS 447000

Gasoline Stations

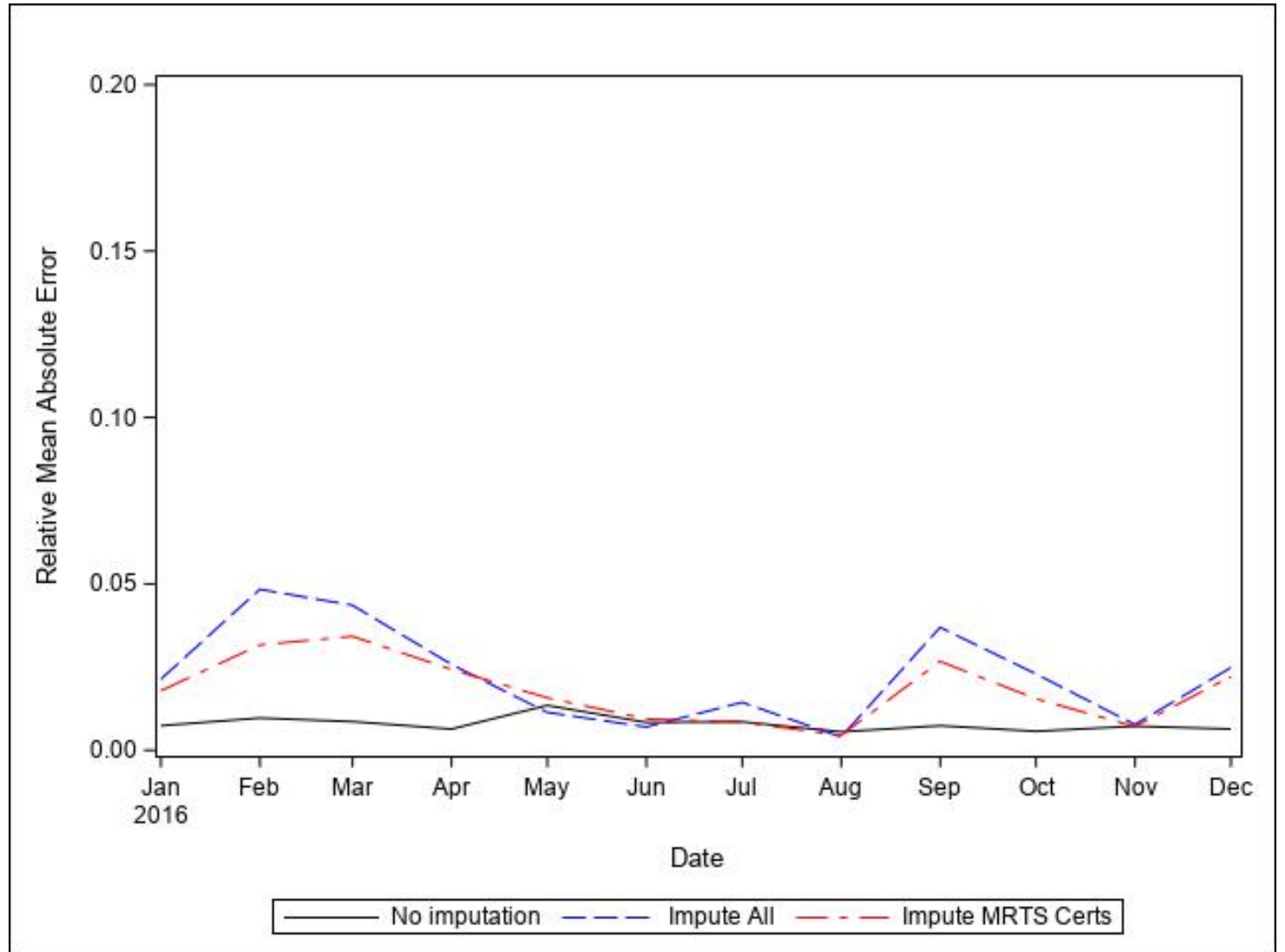


# Effects on Total: No imputation better

Relative MAE

NAICS 447000

Gasoline Stations

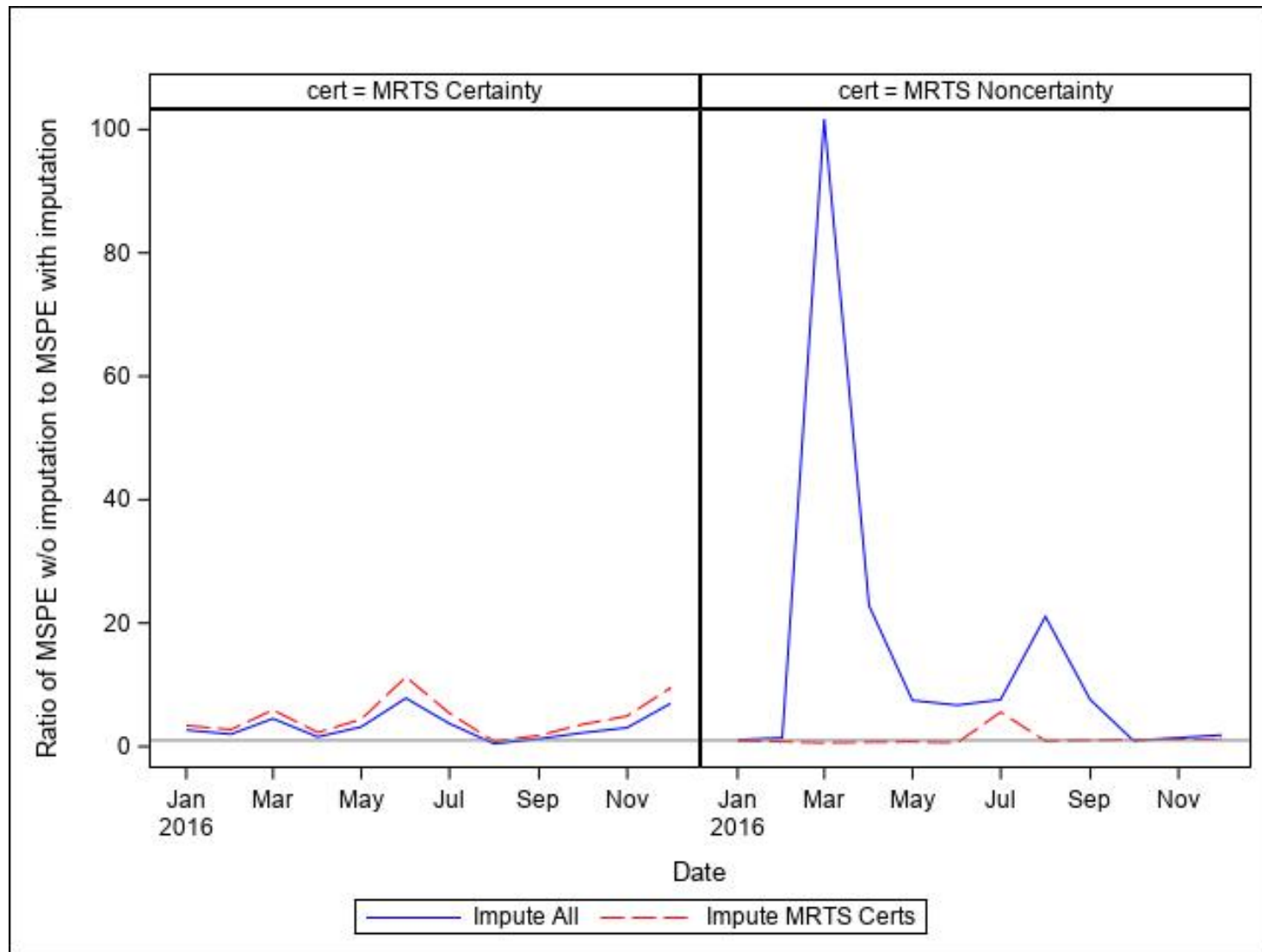


# Unit level prediction errors: Impute all units

$$\frac{MSPE_{noimp}}{MSPE_{impute}}$$

NAICS 448110

Men's Clothing Stores

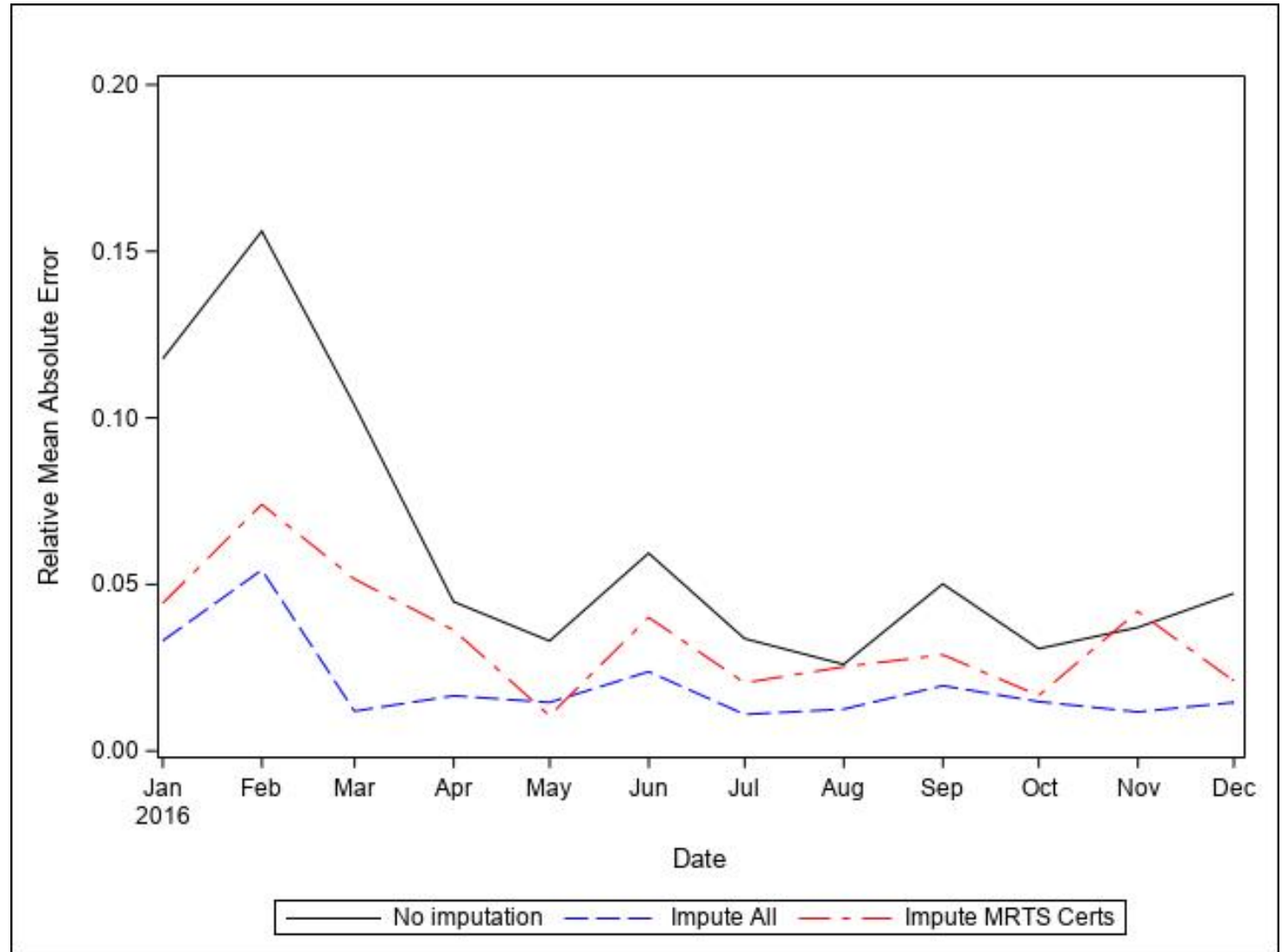


# Effects on Total: Impute all units

Relative MAE

NAICS 448110

Men's Clothing Stores

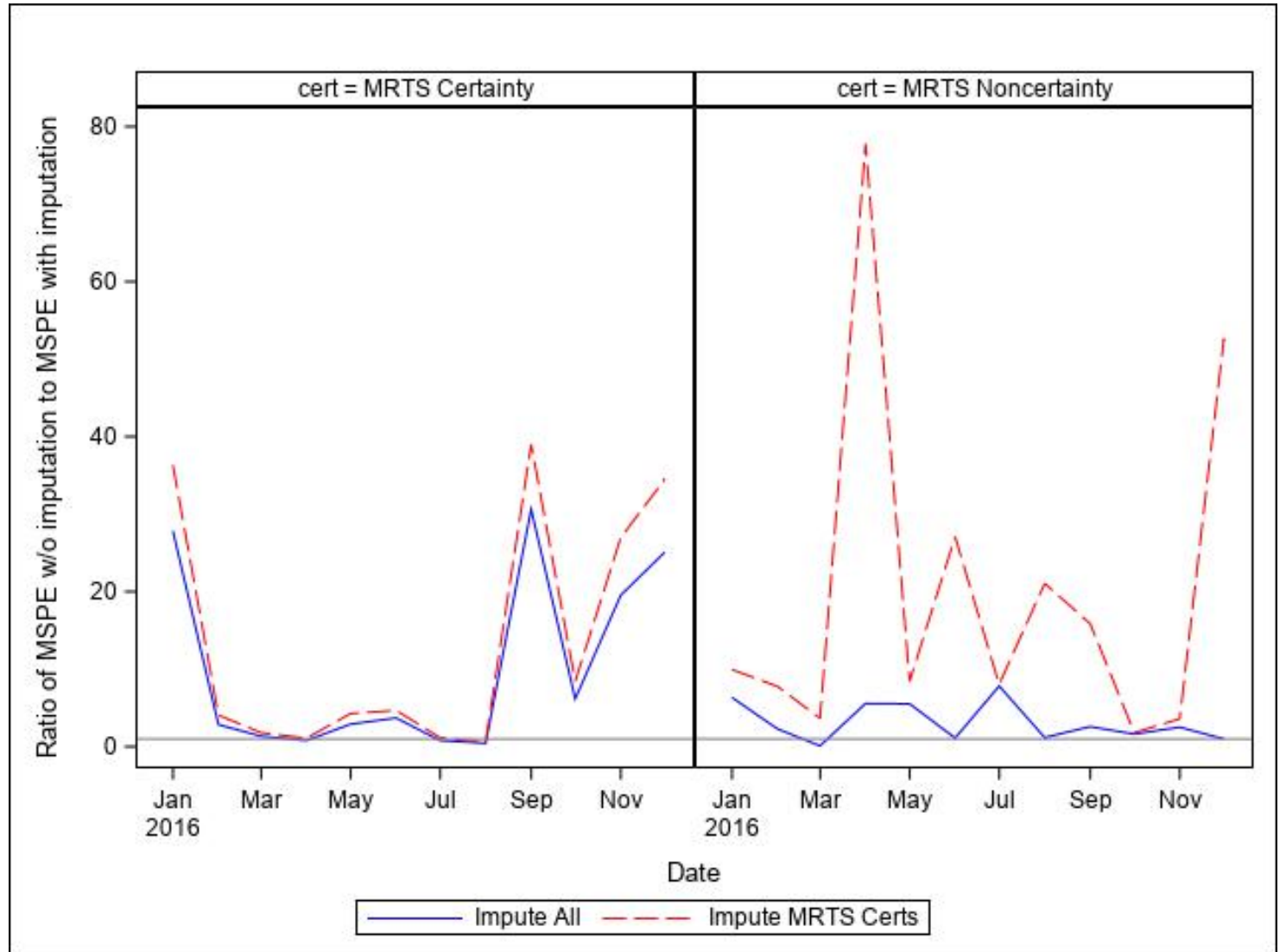


# Unit level prediction errors: Impute MRTS certs

$$\frac{MSPE_{noimp}}{MSPE_{impute}}$$

NAICS 4511X0

Sporting Goods and  
Hobby Stores

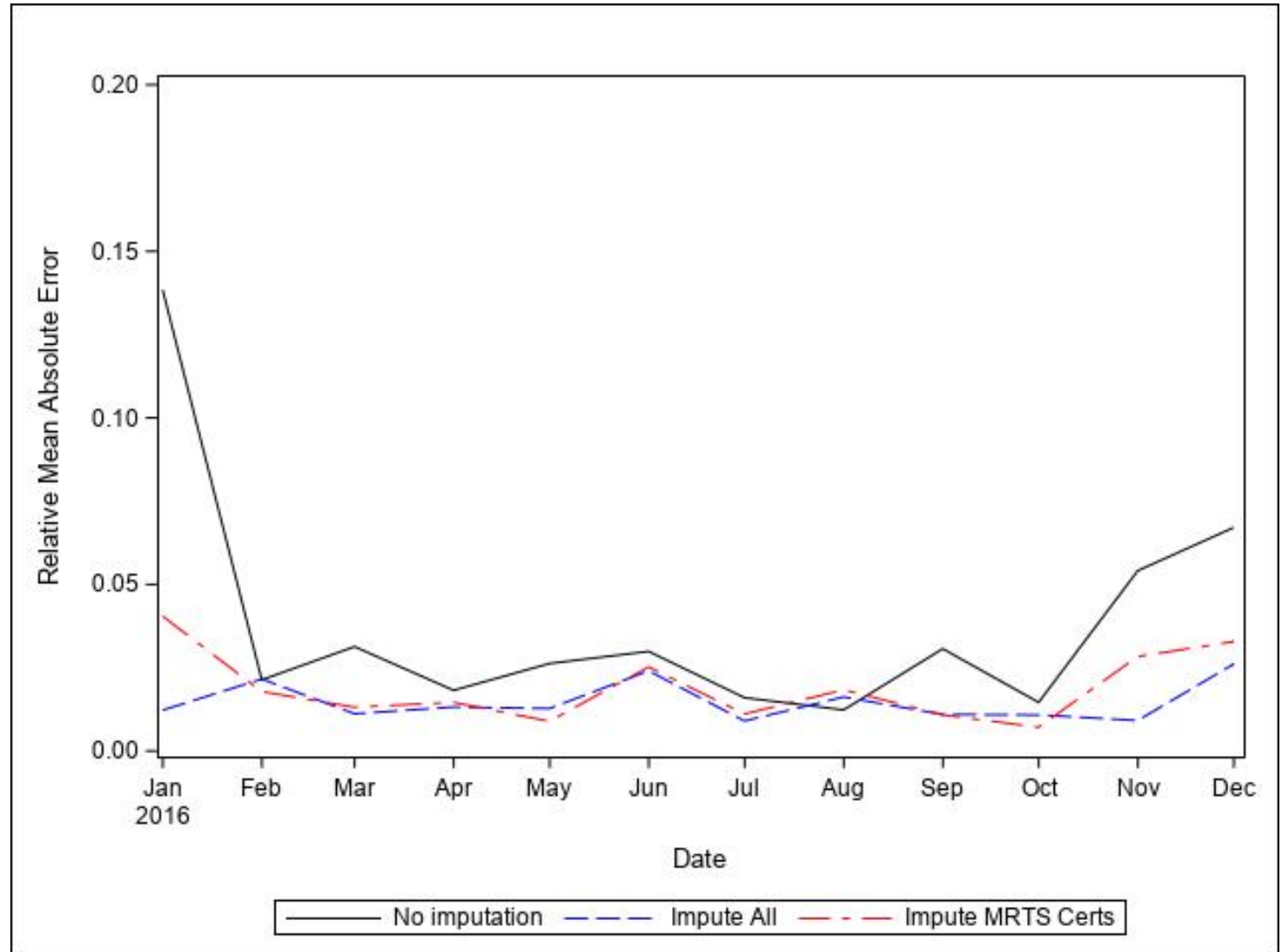


# Effects on Total: Impute MRTS certs only

Relative MAE

NAICS 4511X0

Sporting Goods and  
Hobby Stores



# Conclusions

- RegARIMA forecast imputation reduces MSPE and MAE for many MARTS industries
- However, there are some MARTS industries where the link relative estimator without imputation better accounts for nonresponse
- Most effective for large units: limited use for smaller units
- Imputing only the MRTS certainty units may be enough to see the benefits of imputation and offers consistent methodology throughout the sample lifecycle

# Future Work

- Repeat simulation with response propensities as a function of size
- Refinements to RegARIMA forecast imputation method
  - “Update” one-step ahead forecast with information from current month reported data
  - If we only impute MRTS certainty units, could look at models that use data further in the past
- Incorporating uncertainty due to imputation into variance estimates



# Questions?

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