The Effect of Forecasting on X-11 Adjustment Filters

Brian C. Monsell*

Abstract
This research looks into the effect of forecast extension on the overall seasonal adjustment filter. Weights for the overall adjustment filter were generated for series extended by forecasts of the airline model derived with different values of the nonseasonal and seasonal MA parameters. This paper will focus on the adjustment filter for the concurrent adjustment, which refers to the adjustment of the most recent observation in the series. Details on how the filters vary for length of forecast, choice of model parameters, and choice of seasonal filter are given.
Key Words: Concurrent seasonal adjustment, regARIMA forecasting

Disclaimer

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1. Introduction

Arising from the controversy over the affect of the Great Recession on the seasonal adjustments released by statistical agencies, a question asked by many observers was how much data do the X-11 filters need to perform a seasonal adjustment. One factor not usually taken into consideration by seasonal adjustment experts was the use of forecast extension in official seasonal adjustment. It is a common practice in seasonal adjustment to extend the series with forecasts; its benefits, both theoretical and empirical, have been documented in many publications, including Geweke (1978), Dagum (1988) and Bobbitt and Otto (1990) and the articles referenced in these papers. Forecasts are linear combinations of the original data; does this effect the reach of the filter resulting from combination of the X-11 and forecast formulas?

To answer this, this paper will generate the concurrent seasonal adjustment filter for all combinations of seasonal and trend filters commonly used in X-13ARIMA-SEATS when extending the series with one of the most common ARIMA models, the airline model. We will examine the effect of different model coefficient choices and length of forecast, as well as choices of the trend and seasonal filter.

2. Methodology

Rather than compute the weights for the filter algebraically as in Bell and Monsell (1992), we will use a ”brute force” method to derive the adjustment filters needed for this study. The filter coefficients are computed using the impulse response method (see Appendix B of Findley and Martin (2004), page 204 of Bloomfield (2000) or page 49 of Ladiray and Quenneville (2001)), which finds the matrix representation of a linear transformation by applying it to sequences which are the columns of an identity matrix.

*U. S. Census Bureau,4600 Silver Hill Road, Washington, DC 20233, brian.c.monsell@census.gov

We generate 361 indicator monthly time series of length 361, where $Y(i)_t$ is the $t-th$ observation of series i and $Y(i)_t = 1$ only if $t = i$, with $Y(i)_t = 0$ otherwise. Each of these series will be seasonally adjusted using the X-11 module within X-13ARIMA-SEATS (see Census, 2013) using a specific set of seasonal and trend filters. The resulting seasonally adjusted series is saved after each run. In some of the runs, the indicator series were extended with SARIMA model forecasts with fixed ARMA coefficients. To ensure that X-11's extreme value procedure is not used, the sigma limits for all runs were set to be extremely high numbers.

Let w_k be the filter coefficient for the concurrent seasonal adjustment filter for position $k = 0, \ldots, N$, where N+1 is the length of the series. For any set of seasonal adjustment options with or without forecast extension, the filter coefficients of the associated seasonal adjustment filter are

$$w_j = SA(Y(N+1-j))_{N+1},$$

the $N+1$st observation of the seasonal adjustment of $Y(N+1-j)$. That is, for a series $X_1, \ldots, X_{N+1}$, the seasonally adjusted value for $t = N+1$ is given by

$$\sum_{j=0}^{N} w_j X_{N+1-j}.$$

One could similarly generate coefficients for neighboring filters (ie, 1 month after the concurrent adjustment, a year after the concurrent adjustment, etc.); for reasons of space we will only deal with the concurrent filter here.

The concurrent adjustment filters are generated for all combinations of the following options:

- Seasonal filter: 3x1, 3x3, 3x5, and 3x9,
- Trend Filter: 9, 13, and 23 term Henderson filters,
- Forecasts: 0, 12, 24, and 60 months ahead from observation 361.

Thus, the seasonal adjustments for month 361 were obtained for series $Y(i)_t, t = 1, \cdots, 361$ and three forecast extended series, with forecasts through times 361+12, 361+24, and 361+60, respectively. The resulting seasonal adjustments provide the i-th filter coefficient for the four forecast extension cases.

A single SARIMA model type was used to generate the forecast extensions in this study - the airline model, SARIMA $(011)(011)$. The model used for Y_t is $(1-B)(1-B^{12})Y_t = (1-\theta B)(1-\Theta B^{12})a_t$.

The model coefficient pairs specified were all combinations of the following values:

- Nonseasonal MA : 0.3, 0.5, 0.7, 0.9
- Seasonal MA : 0.3, 0.5, 0.7, 0.9

3. Comparisons of Interest

To compare the coefficients for the concurrent adjustment filter for various seasonal adjustment options, we will use graphs to display the filter coefficients. We will compare plots for filters derived from different adjustment options to see how the coefficients of the adjustment filter change according to the:

- length of the forecast extension;
- value of the ARIMA model coefficients; and
- seasonal and trend filter.

One of the filter characteristics we will be examining is how many years of data are needed for each type of seasonal filter.

3.1 Length of Forecasts

Figures 1 and 2 show two plots of the concurrent adjustment filter coefficient values for the data at lags $361 - k, k = 0, \cdots, 24$. The adjustments were performed using the 3x5 seasonal and 13-term Henderson filters, with 0, 12, 24, and 60 months of ARIMA forecasts extended to the series.

The most apparent characteristic of the filter coefficients is that the spread between the different coefficients is most pronounced at position 0 and at multiples of 12 thereafter. Often the magnitudes of the filter coefficients are larger for the adjustment without forecast extension, and they get smaller as the forecast interval length increases. The spread between the filter coefficients gets smaller as the value of the seasonal MA Θ parameter increases.

For later filter coefficients at seasonal lags, the relative positioning of the filter coefficients changes. This can be influenced by the value of the seasonal MA parameter - as the seasonal MA parameter gets larger, the filter coefficients often get closer to zero, compensating for the increase in the value of the filter coefficient at lag 0.

It can be difficult to examine the coefficients when the coefficient at k=0 is so much larger than the others. If this coefficient is suppressed in the plot, we can look for more subtle patterns in the other coefficients. Figures 3 and 4 show the filter coefficient values shown in figures 1 and 2, respectively, without the coefficient at $k = 0$ and with the number of lags shown extended to $k = 60$.

Here it is easier to see that the filter coefficients for the adjustments with forecast extension are smoother than those for the adjustments without forecast extension, and the weights get smoother as the number of the forecasts used to extend the series increases. The filter coefficients also seem to be converging to zero as k increases, except at seasonal lags, whereas the weights seem to be converging to a non-zero number, particularly for the larger seasonal MA values.

3.2 ARIMA Seasonal Moving Average Coefficient

Figures 5 and 6 show two plots of the concurrent adjustment filter coefficient values for adjustments using 3x5 seasonal and 13-term Henderson filters for data at lags $361 - k, k = 0, \cdots, 24$, where the forecasts are generated with airline models with different ARIMA seasonal moving averages. The size of the filter coefficient associated with the concurrent observation is directly related to the size of the seasonal MA parameter in the SARIMA model, and the spread between the coefficients for the different adjustments gets larger as the length of the forecast extension gets longer.

As in section 3.1, figures 7 and 8 suppress the filter coefficient at $k = 0$ to show more details for other lags. We can better see the kink in the pattern between lags 1 and 11. This kink gets less pronounced as the value of the seasonal MA parameter increases. In addition, as the number of forecasts increases, the difference between filter coefficients generated by different values of the seasonal MA coefficient becomes more pronounced (although still quite small).

At the seasonal lags $(12, 24, \cdots)$, the filter coefficients converge at different rates, with the filter coefficients for $\Theta = 0.9$ being the slowest to converge. The filter coefficients also seem to converge slower as the number of forecasts used in the forecast extension increases.

3.3 Nonseasonal Moving Average coefficient

For the **nonseasonal MA parameter**, the biggest spread in the coefficients is at $k = 0$. Otherwise, there is not much of a difference between the coefficients; the only discernable difference in the coefficients is at $k = 1$ and $k = 13$ for the case when $\theta = 0.9$. Figure 9 shows an example of how little spread there is between the filter coefficients for different values of the nonseasonal MA parameter.

3.4 Henderson trend filter

The choice of the **Henderson trend filter** does not seem to affect the concurrent adjustment filter very much - there is little discernable difference in the coefficients in the plots, as seen in figure 10.

3.5 Seasonal filter

Figures 11 and 12 show two plots of the concurrent adjustment filter coefficient values for adjustments using different seasonal filters where the adjustments used to generate the filter coefficients were calculated using two years of forecasts. For the coefficient related to the concurrent observation, the size of the filter coefficient is related to the length of the seasonal filter used in the seasonal adjustment. The size of the filter coefficient is also related to the size of the seasonal MA parameter used in forecast extension.

In figures 13 and 14, the filter coefficients for the different seasonal filters converge at different rates, as one would expect. The rate of convergence is related to the length of the seasonal filter used in the adjustment. While the nonseasonal lags seem to converge to zero, the seasonal filter coefficients seem to be converging to a very small non-zero value, which gets larger as the size of the seasonal MA parameter used to generate the forecasts increases. This value is very small but persistent. If we consider this value to be a baseline for the seasonal filter coefficients, the 3x1 and 3x3 filters converge at about 36 months, the 3x5 filter converges at about 48 months, and the 3x9 filter converges at about 72 months. This corresponds to where the values of the coefficients seem to go to zero for the plots where $\Theta = 0.3$ in figure 13.

Figures 15 and 16 are the same type of plot, except the forecast extension is done for 5 years of forecasts. Increasing the number of forecasts had little effect on how fast the filter coefficients were converging - the value of the seasonal MA coefficient has a far greater effect than lengthening the forecasts.

As the value of the seasonal MA parameter increases, the past-value weights of the forecasting function decay much more slowly (see equation 3.23b of Findley, Pötscher, and Wei (2004)), and this is having an effect on the concurrent adjustment filter.

To get a more detailed view of the filter coefficients at seasonal frequencies, table 1 shows the filter coefficients for the concurrent adjustment filters for the four seasonal filters shown in figures 13 and 14. The filter coefficients are converging to zero more slowly for the case where $\Theta = 0.9$, especially for the 3x9 seasonal filter. Similarly, table 2 shows the filter coefficients for the concurrent adjustment filters for the four seasonal filters shown in figures 15 and 16. Again, the filter coefficients are converging slower for the case where $\Theta = 0.9$, and comparing these figures with those in table 1, the coefficients are slightly

Table 1: **Concurrent adjustment filter coefficient for different seasonal filters, 13-term Henderson, 2 years of forecasts**

k	$\theta = 0.7, \Theta = 0.3$				$\theta = 0.7, \Theta = 0.9$			
	3x1	3x3	3x5	3x9	3x1	3x3	3x5	3x9
0	0.5605	0.5691	0.6478	0.7139	0.7136	0.7148	0.8009	0.8520
12	-0.4072	-0.3230	-0.3108	-0.2246	-0.3202	-0.2417	-0.2394	-0.1657
24	-0.0539	-0.1383	-0.1658	-0.1557	-0.0465	-0.1316	-0.1617	-0.1532
36	-0.0113	-0.0181	-0.0732	-0.1241	-0.0268	-0.0328	-0.0882	-0.1375
48	-0.0033	-0.0051	-0.0100	-0.0784	-0.0239	-0.0245	-0.0291	-0.0951
60	-0.0010	-0.0010	-0.0037	-0.0339	-0.0215	-0.0203	-0.0226	-0.0503
72	-0.0003	-0.0003	-0.0010	-0.0066	-0.0194	-0.0183	-0.0185	-0.0219
84	-0.0001	-0.0001	-0.0001	-0.0041	-0.0174	-0.0165	-0.0160	-0.0180
96	0.0000	0.0000	0.0000	-0.0022	-0.0157	-0.0148	-0.0144	-0.0147
108	0.0000	0.0000	0.0000	-0.0008	-0.0142	-0.0134	-0.0130	-0.0122
120	0.0000	0.0000	0.0000	-0.0002	-0.0128	-0.0121	-0.0118	-0.0104
132	0.0000	0.0000	0.0000	0.0000	-0.0116	-0.0109	-0.0106	-0.0092
144	0.0000	0.0000	0.0000	0.0000	-0.0104	-0.0098	-0.0096	-0.0084
156	0.0000	0.0000	0.0000	0.0000	-0.0094	-0.0089	-0.0087	-0.0076
168	0.0000	0.0000	0.0000	0.0000	-0.0085	-0.0081	-0.0078	-0.0068
180	0.0000	0.0000	0.0000	0.0000	-0.0077	-0.0073	-0.0071	-0.0062

larger for the coefficients at $k = 0$, and slightly smaller than those at the other seasonal frequencies.

On a minor note, the shape of the 3x1 filter coefficients seem different, particularly leading up to the seasonal lags, than the coefficients from the other seasonal filters.

4. Conclusions

We have shown that the SARIMA model used for forecast extension can have an effect on how fast the seasonal filter coefficients converge to zero. We have also seen that the level of the seasonal moving average and the length of the forecast extension affect the level and smoothness of the seasonal filter coefficients.

5. Acknowledgments

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Table 2: **Concurrent adjustment filter coefficient for different seasonal filters, 13-term Henderson, 5 years of forecasts**

k	$\theta = 0.7, \Theta = 0.3$				$\theta = 0.7, \Theta = 0.9$			
	3x1	3x3	3x5	3x9	3x1	3x3	3x5	3x9
0	0.5610	0.5687	0.6304	0.6724	0.7150	0.7270	0.8176	0.8784
12	-0.4076	-0.3191	-0.2990	-0.2259	-0.3204	-0.2322	-0.2119	-0.1394
24	-0.0540	-0.1408	-0.1580	-0.1363	-0.0467	-0.1337	-0.1531	-0.1329
36	-0.0113	-0.0188	-0.0746	-0.1075	-0.0269	-0.0347	-0.0929	-0.1274
48	-0.0033	-0.0053	-0.0010	-0.0693	-0.0239	-0.0263	-0.0337	-0.0942
60	-0.0010	-0.0010	-0.0038	-0.0368	-0.0216	-0.0219	-0.0269	-0.0612
72	-0.0003	-0.0002	-0.0012	-0.0061	-0.0194	-0.0197	-0.0225	-0.0287
84	-0.0001	-0.0001	-0.0001	-0.0034	-0.0175	-0.0178	-0.0196	-0.0246
96	0.0000	0.0000	0.0000	-0.0022	-0.0158	-0.0160	-0.0177	-0.0208
108	0.0000	0.0000	0.0000	-0.0009	-0.0142	-0.0144	-0.0159	-0.0177
120	0.0000	0.0000	0.0000	-0.0002	-0.0128	-0.0130	-0.0144	-0.0154
132	0.0000	0.0000	0.0000	0.0000	-0.0116	-0.0118	-0.0130	-0.0137
144	0.0000	0.0000	0.0000	0.0000	-0.0105	-0.0106	-0.0117	-0.0124
156	0.0000	0.0000	0.0000	0.0000	-0.0095	-0.0096	-0.0106	-0.0112
168	0.0000	0.0000	0.0000	0.0000	-0.0086	-0.0087	-0.0096	-0.0101
180	0.0000	0.0000	0.0000	0.0000	-0.0078	-0.0079	-0.0087	-0.0092

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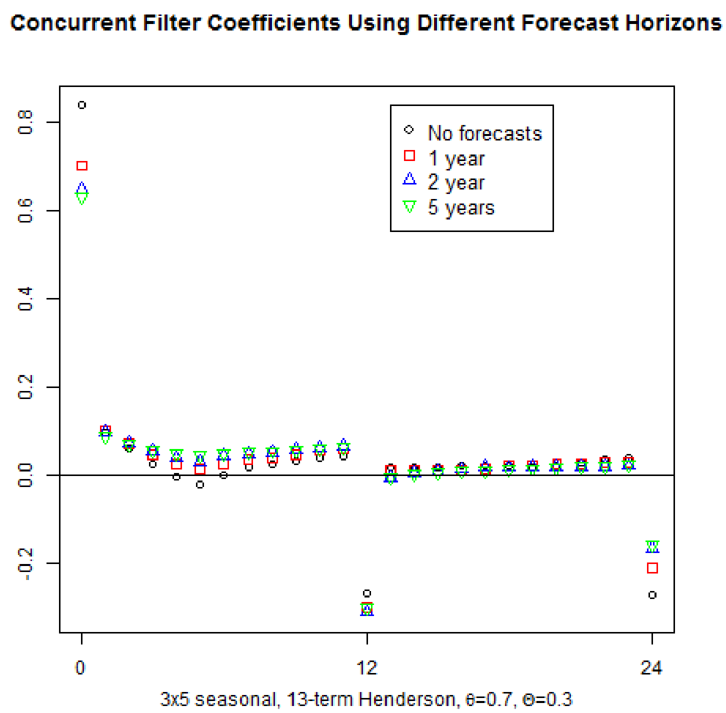

Figure 1: Filter coefficients for concurrent seasonal adjustment using 3x5 seasonal and 13-term Henderson filter for different forecast horizons from airline models, $\theta = 0.7, \Theta = 0.3$

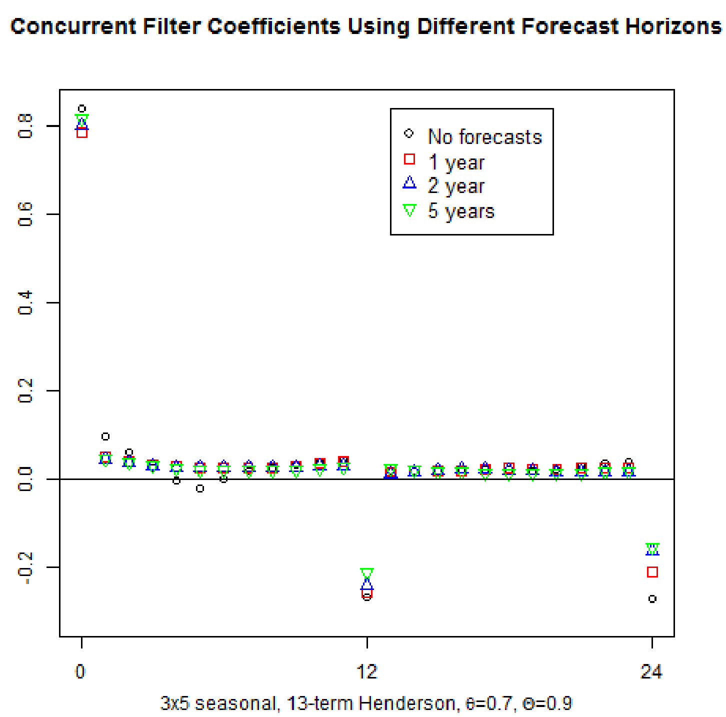

Figure 2: Filter coefficients for concurrent seasonal adjustment using 3x5 seasonal and 13-term Henderson filter for different forecast horizons from airline models, $\theta = 0.7, \Theta = 0.9$

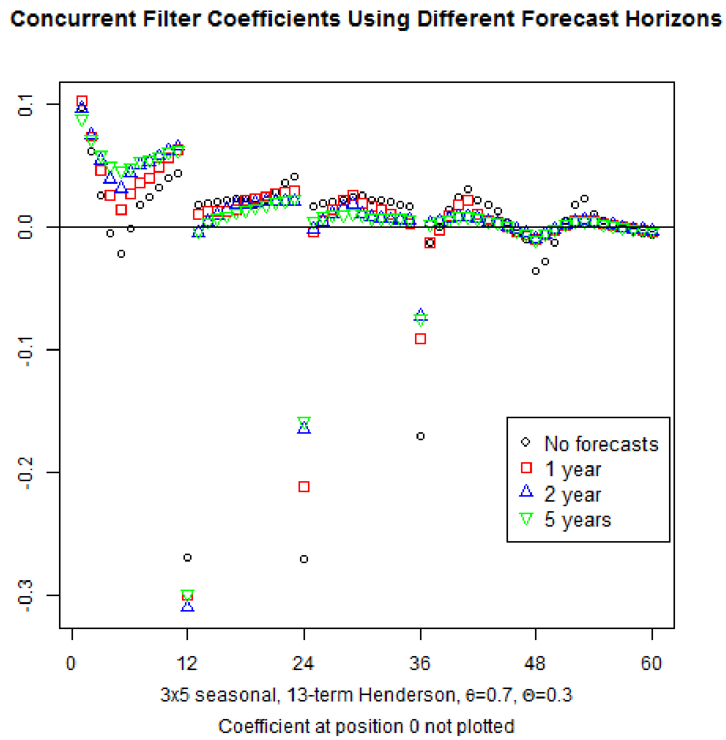

Figure 3: Filter coefficients for concurrent seasonal adjustment using 3x5 seasonal and 13-term Henderson filter for different forecast horizons from airline models, $\theta = 0.7$, $\Theta = 0.3$, first coefficient suppressed

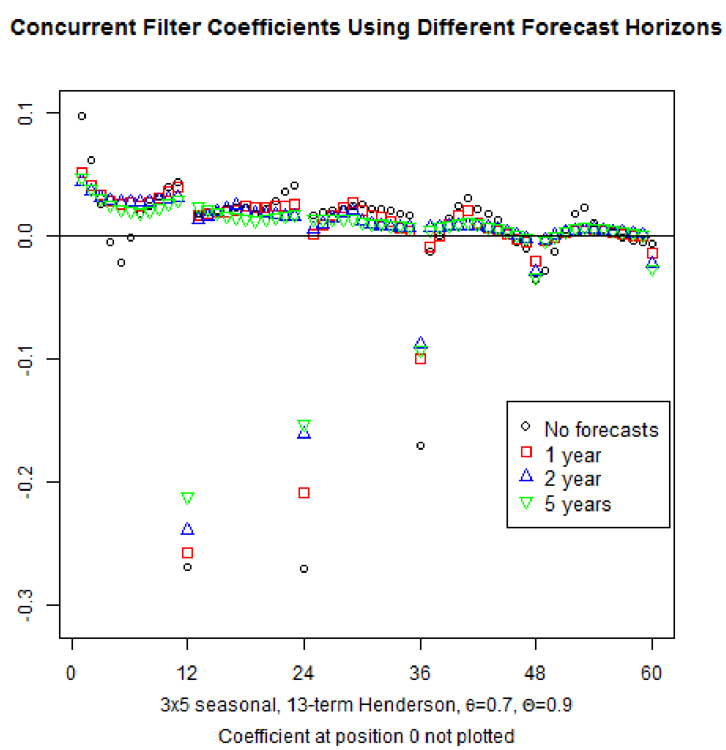

Figure 4: Filter coefficients for concurrent seasonal adjustment using 3x5 seasonal and 13-term Henderson filter for different forecast horizons from airline models, $\theta = 0.7$, $\Theta = 0.9$, first coefficient suppressed

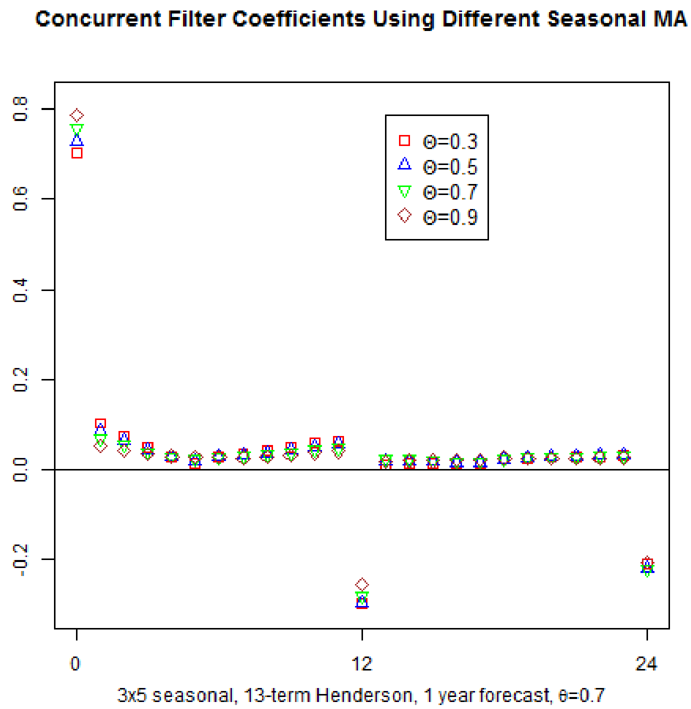

Figure 5: Filter coefficients for concurrent seasonal adjustment (3x5 seasonal, 13-term Henderson filter) for airline models with different Θ, series extended with 12 months of forecasts

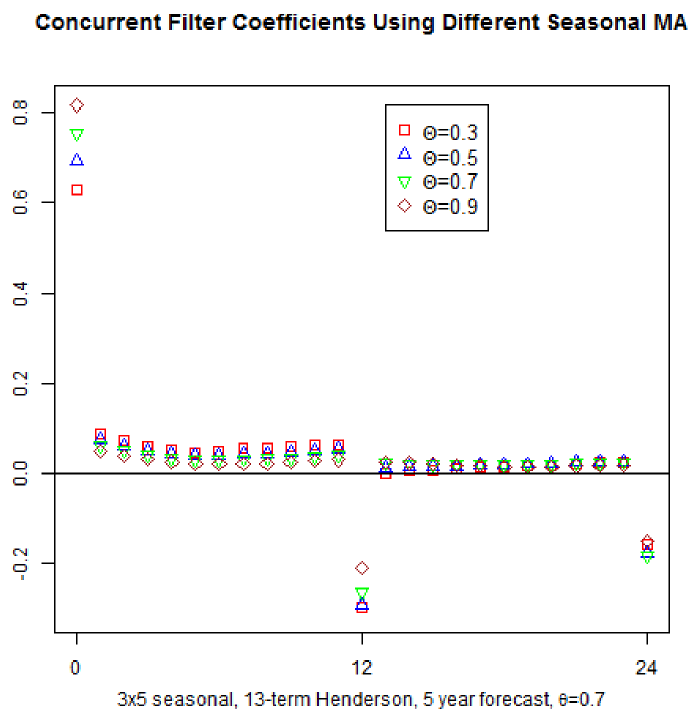

Figure 6: Filter coefficients for concurrent seasonal adjustment (3x5 seasonal, 13-term Henderson filter) for airline models with different Θ, series extended with 60 months of forecasts

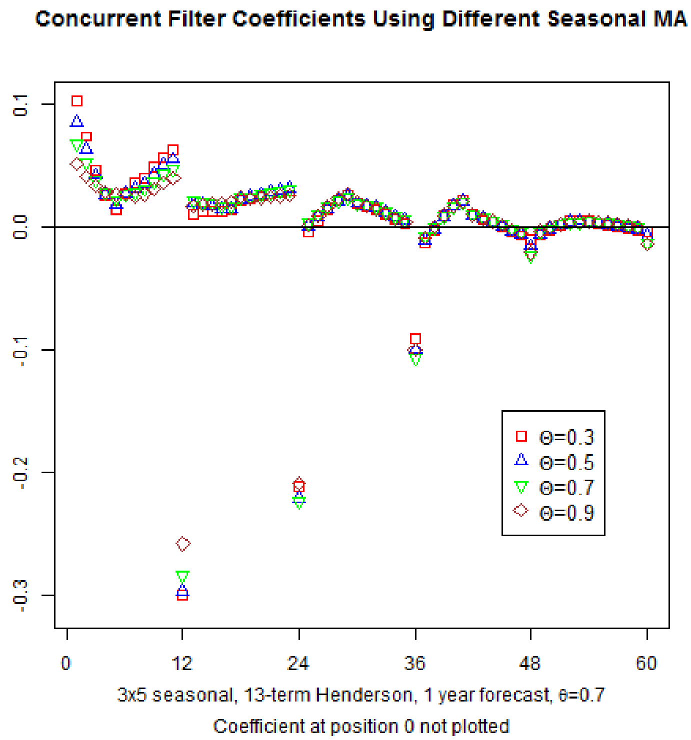

Figure 7: Filter coefficients for concurrent seasonal adjustment (3x5 seasonal, 13-term Henderson filter) for airline models with different Θ, series extended with 12 months of forecasts, first coefficient suppressed

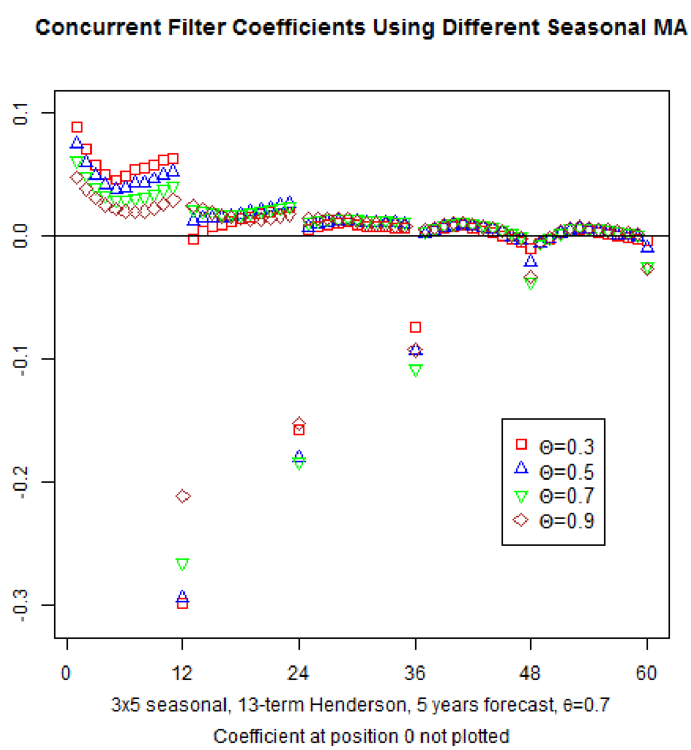

Figure 8: Filter coefficients for concurrent seasonal adjustment (3x5 seasonal, 13-term Henderson filter) for airline models with different Θ, series extended with 60 months of forecasts, first coefficient suppressed

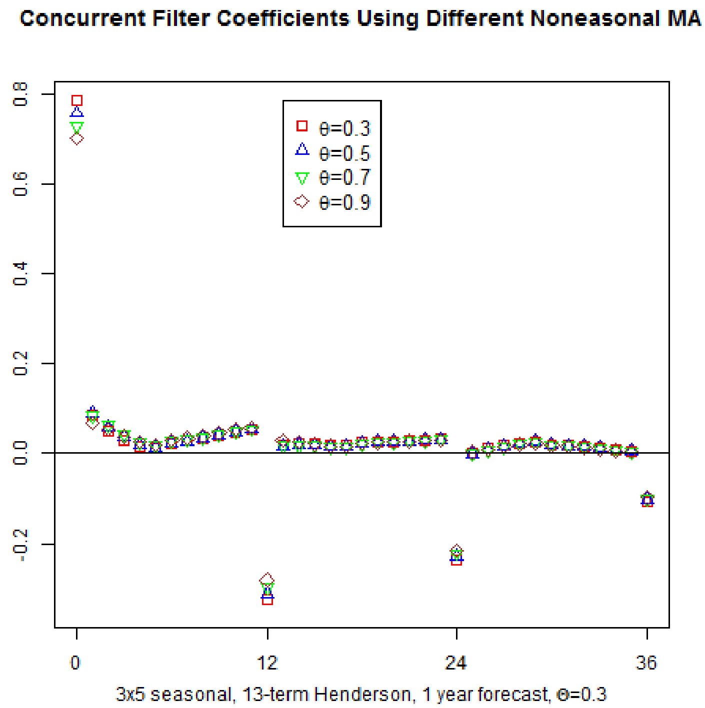

Figure 9: Filter coefficients for concurrent seasonal adjustment (3x5 seasonal, 13-term Henderson filter) for airline models with different θ, series extended with 12 months of forecasts

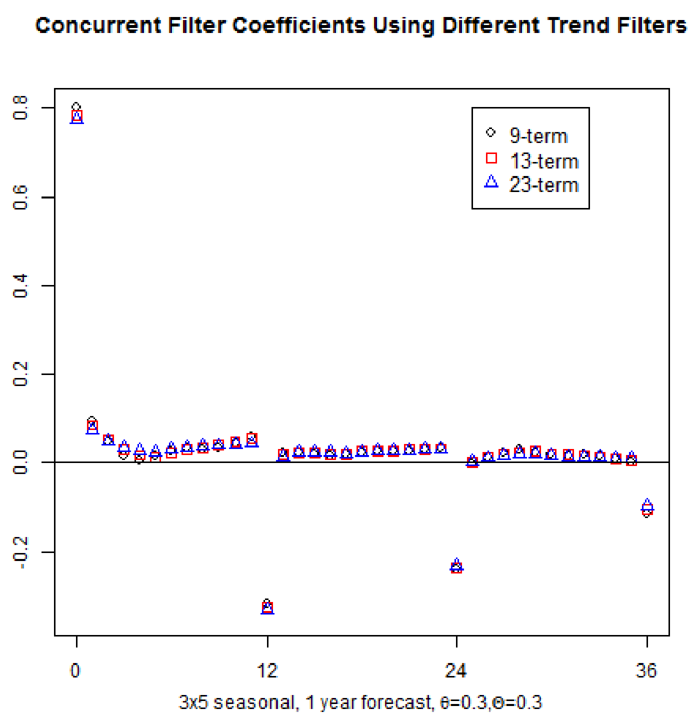

Figure 10: Filter coefficients for concurrent seasonal adjustment (3x5 seasonal, 1 years of forecast from an airline model with $\theta = 0.3, \Theta = 0.3$) for Henderson trend filter choices

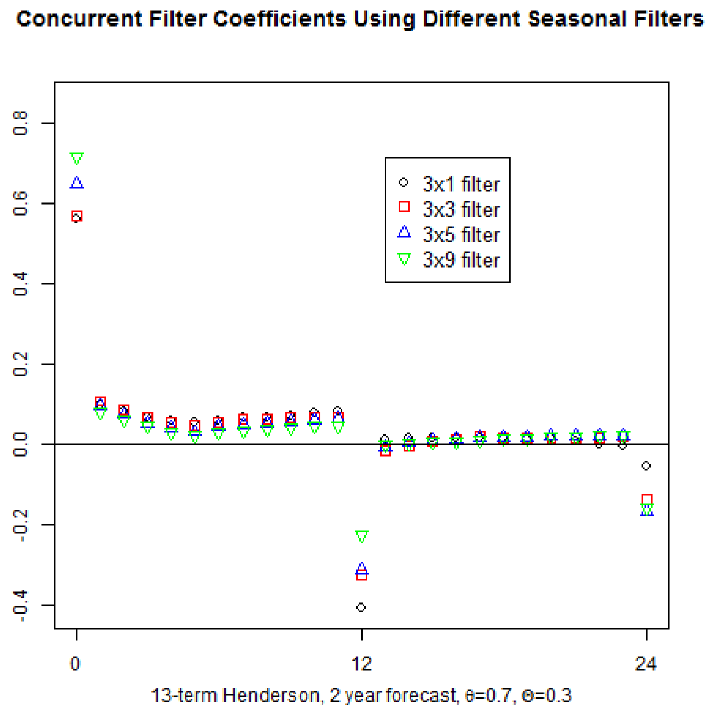

Figure 11: Filter coefficients for concurrent seasonal adjustment (13-term Henderson filter, 2 years of forecasts from an airline model with $\theta = 0.7, \Theta = 0.3$) for different seasonal filter choices

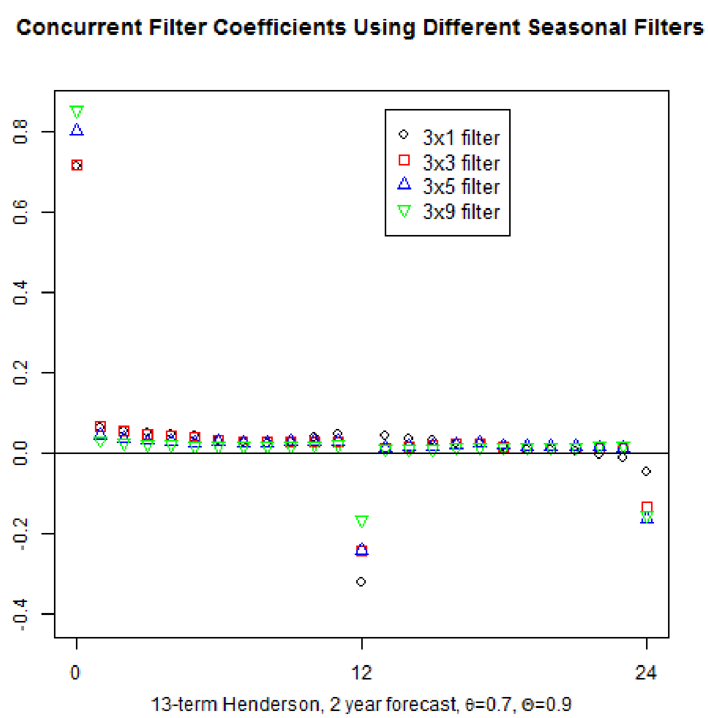

Figure 12: Filter coefficients for concurrent seasonal adjustment (13-term Henderson filter, 2 years of forecasts from an airline model with $\theta = 0.7, \Theta = 0.9$) for different seasonal filter choices

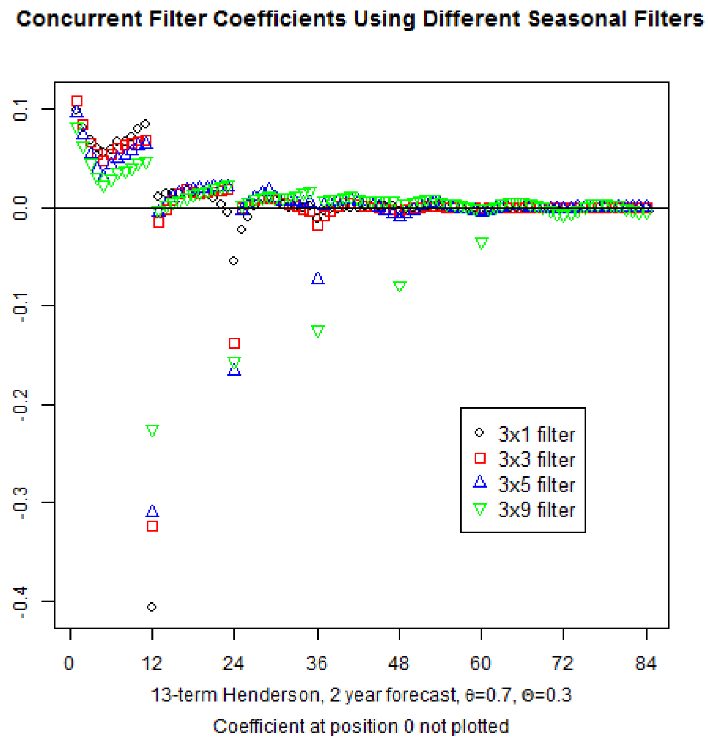

Figure 13: Filter coefficients for concurrent seasonal adjustment (13-term Henderson filter, 2 years of forecasts from an airline model with $\theta = 0.7, \Theta = 0.3$) for different seasonal filter choices, first coefficient suppressed

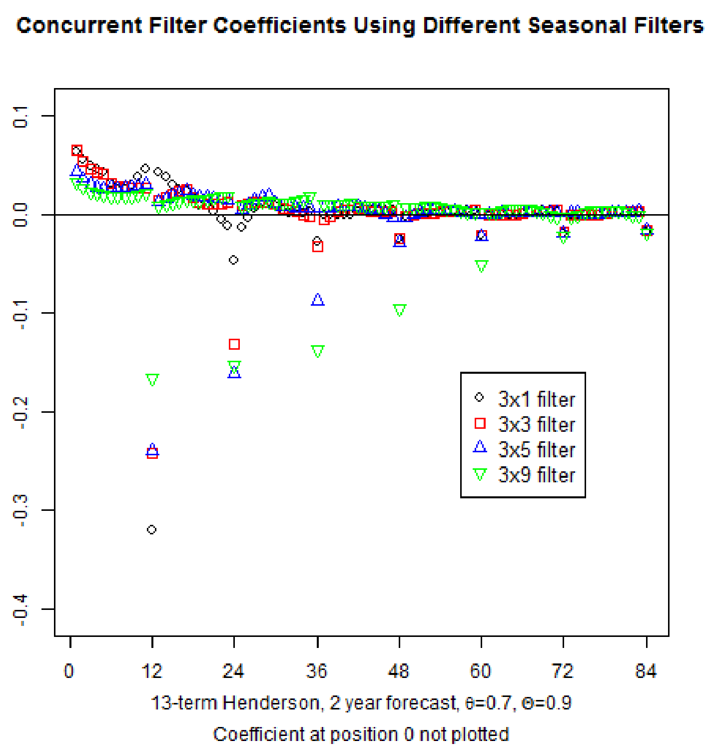

Figure 14: Filter coefficients for concurrent seasonal adjustment (13-term Henderson filter, 2 years of forecasts from an airline model with $\theta = 0.7, \Theta = 0.9$) for different seasonal filter choices, first coefficient suppressed

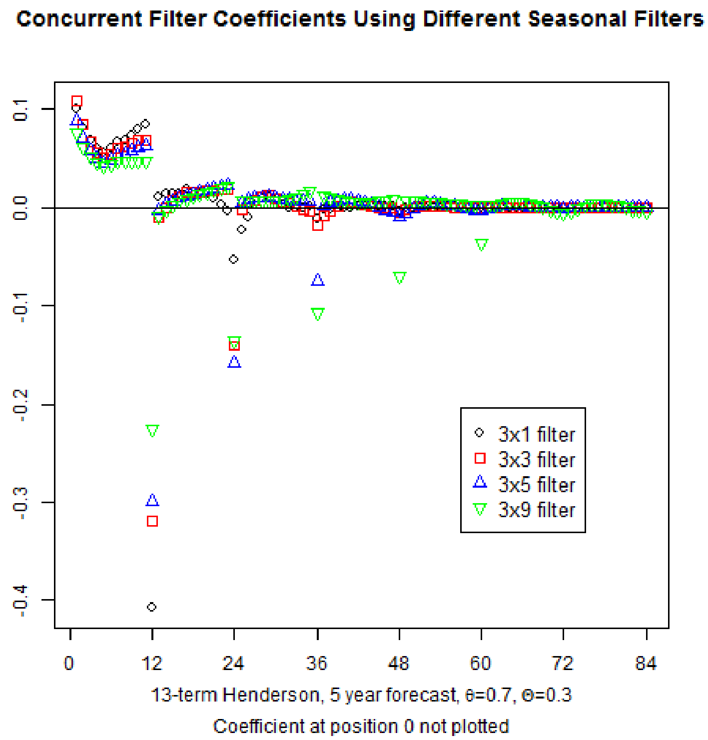

Figure 15: Filter coefficients for concurrent seasonal adjustment (13-term Henderson filter, 5 years of forecasts from an airline model with $\theta = 0.7, \Theta = 0.3$) for different seasonal filter choices, first coefficient suppressed

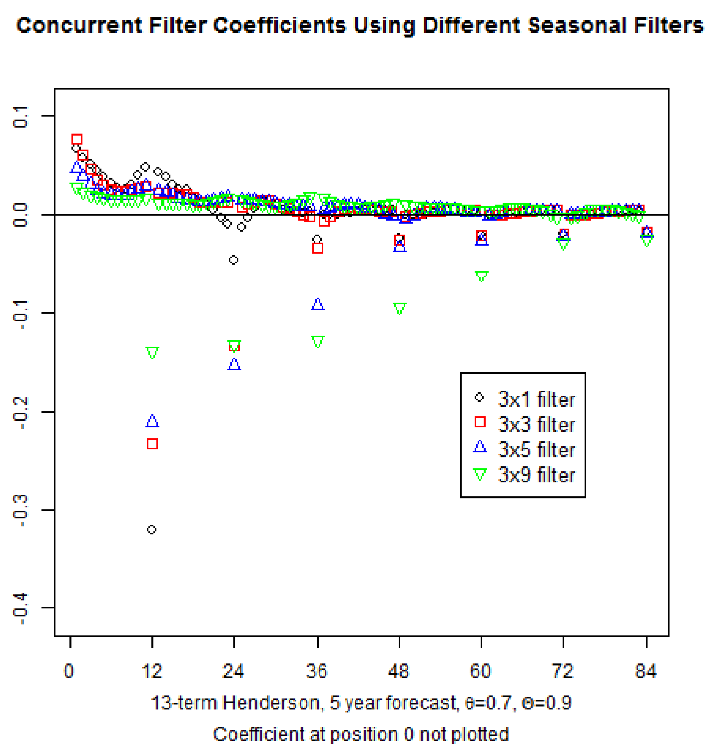

Figure 16: Filter coefficients for concurrent seasonal adjustment (13-term Henderson filter, 5 years of forecasts from an airline model with $\theta = 0.7, \Theta = 0.9$) for different seasonal filter choices, first coefficient suppressed