Methodology for Testing for a Rise in State Child Poverty Rates of Five Percent or Greater:

2006 to 2007

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1 Background

The U.S. Census Bureau's Small Area Estimates Branch annually provides the Administration for Children and Families in the Department of Health and Human Services (HHS) with model-based estimates of the number of children under age 18 in poverty. These estimates are used to determine if any states had an increase of 5 percent or greater in child poverty rate between two consecutive years. This document addresses change between 2006 and 2007.

The data presented help identify states for which the following equivalent statements are true:

$$\frac{(\text{2007 Poverty Rate}) - (\text{2006 Poverty Rate})}{\text{2006 Poverty Rate}} \geq 0.05$$

$$(\text{2007 Poverty Rate}) - (\text{2006 Poverty Rate}) \geq 0.05 \cdot (\text{2006 Poverty Rate})$$

$$(\text{2007 Poverty Rate}) - 1.05 \cdot (\text{2006 Poverty Rate}) \geq 0$$

The poverty estimates used in this analysis are from the Small Area Income and Poverty Estimates (SAIPE) program. The SAIPE program produces model-based estimates of poverty that combine estimates of poverty from the American Community Survey (ACS) with regression predictions of poverty based on administrative records, intercensal population estimates, and Census 2000 data. The modeling techniques allow SAIPE to produce annual estimates of child poverty for all school districts and all counties, regardless of population size. For state estimates, like those in this report, the benefits of the additional data sources vary by state. For states with large populations, the SAIPE estimate tends to be close to the corresponding ACS direct estimate. For states with small populations, the SAIPE estimate tends to have a lower variance than the ACS direct estimate (giving a more precise estimate), and may be to some extent different from the ACS direct estimate.

Documentation of the methods used to produce SAIPE estimates for 2006 and 2007 is available on the SAIPE program's website, http://www.census.gov/did/www/saipe/methods/statecounty/20062007state.html.

Within this document, "change estimate" refers to the 2007 poverty rate estimate for children under age 18 minus the 2006 poverty rate estimate for children under age 18. Accordingly, "change variance estimate" refers to the variance of this quantity, and "*z*-statistic" refers to the ratio of "change estimate" to the square root of "change variance estimate." Terminology for the 5 percent or greater data corresponds: "1.05 change estimate" refers to the 2007 poverty rate estimate for children under age 18 minus 1.05 times the 2006 poverty rate estimate for children under age 18, "1.05 change variance estimate" refers to the variance of this quantity, and "1.05 *z*-statistic" refers to the ratio of "1.05 change estimate" to the square root of "1.05 change variance estimate."

Section 2 below describes the type of hypothesis tests used to assess year-to-year change in the child poverty rates. *Section 3* discusses data sources for the SAIPE estimates, and *Section 4* presents the results for the hypothesis tests. *Section 5* presents mathematical details behind the poverty rate estimation and change variance estimation.

State poverty estimates, standard errors, and z-statistics are shown in *Table 1* and *Table 2* at the end of the document.

2 Hypothesis Tests

State-level estimates of child poverty rates from the SAIPE program can be used to test whether there is statistically significant evidence that the child poverty rate has increased by 5 percent or more. The 1.05 *z*-statistics are created for the one-tailed hypothesis test as follows:

Null Hypothesis: Poverty rate has *not* increased by 5 percent or greater

$$(2007 \text{ Poverty Rate}) - 1.05 \cdot (2006 \text{ Poverty Rate}) < 0$$

Alternative Hypothesis: Poverty rate has increased by 5 percent or greater

$$(2007 \text{ Poverty Rate}) - 1.05 \cdot (2006 \text{ Poverty Rate}) \geq 0$$

Test Statistic (the 1.05 *z*-statistic):

$$z = \frac{((2007 \text{ Poverty Rate Estimate}) - 1.05 \cdot (2006 \text{ Poverty Rate Estimate}))}{\sqrt{\text{Var}((2007 \text{ Poverty Rate Estimate}) - 1.05 \cdot (2006 \text{ Poverty Rate Estimate}))}} \tag{1}$$

The 1.05 *z*-statistic in formula (1) can be compared to critical values from the standard normal distribution. Evaluating the denominator of formula (1) requires variances and covariances of the SAIPE poverty rate estimates. The steps involved in estimating these variances and covariances are described in Sections 5.3 and 5.4.

A single one-tailed test would be appropriate to test for an increase of 5 percent or greater in a particular state. However, since we are testing for an increase of 5 percent or greater in all 50 states and the District of Columbia, applying one-tailed tests separately for each state would be inappropriate. In particular, if no state had an increase of 5 percent or greater and we performed this test separately for each state, then the probability we would conclude one or more states had a 5 percent or greater increase may be larger than the stated significance level. This is referred to as the problem of "multiple comparisons."

In order to test whether there has been a child poverty rate increase of 5 percent or greater in any of the 50 states and the District of Columbia, we follow the Bonferroni approach. The Bonferroni approach addresses the problem of multiple comparisons by using a critical value such that, if all the null hypotheses for a set of tests were true, the probability that one or more of these tests would yield a statistically significant result will be no larger than the specified significance level.

3 Source Data for SAIPE Estimates

The model-based estimates from the SAIPE program employ both direct survey-based estimates from the ACS and regression predictions based on administrative records, intercensal population estimates and Census 2000 data. The direct survey-based estimates of poverty are combined with the regression predictions of poverty using Bayesian techniques. The Bayesian techniques weight the contribution of the two components (direct estimates and regression predictions) on the basis of their relative precision. This is done separately for each year. The modeling details are discussed in Section 5.1.

The ACS is a nationwide survey designed to provide communities with reliable and timely demographic, social, economic, and housing data every year. After several years of ACS demonstration surveys, in August 2006, estimates from the 2005 full-production ACS were released. Starting with the 2006 ACS data (released in August 2007), group quarters populations were also included in the ACS. The full-production ACS has a sample size of roughly 3 million addresses annually, and the sample is selected from all counties and county-equivalents in the United States and from all municipios in Puerto Rico. Results from the ACS are discussed in the annual report *Income, Earnings, and Poverty Data From the 2007 American Community Survey* (Bishaw and Semega, 2008).

The ACS collects data throughout the year on an on-going, monthly basis and asks for a respondent's income over the "past 12 months." As a result, adjacent years of ACS will have some reference months in common. Hence, comparing the 2007 ACS estimates with the 2006 ACS estimates directly is not an exact comparison of the economic conditions in calendar year 2007 with those in calendar year 2006. For a discussion of this and related issues, see Hogan (2008). Also, substantial year-to-year changes in the ACS group quarters estimates can impact ACS estimates of total population characteristics. However, for estimates of poverty rates for children under age 18 these concerns are minimal.

For the 2004 SAIPE estimates and prior years, the SAIPE program modeled poverty estimates from the Annual Social and Economic Supplement to the Current Population Survey (CPS ASEC). Starting with the 2005 ACS estimates, the Census Bureau recommends using state-level ACS estimates as the official direct state estimates. As a result, the SAIPE program switched to modeling ACS state poverty estimates for the 2005 SAIPE estimates. Documentation for this change is available on the SAIPE program webpage, http://www.census.gov/did/www/saipe/methods/05change.html.

While this document relates specifically to state-level estimates only, the CPS ASEC remains the official source of the *national* poverty estimates that are calculated in accordance with the Office of Management and Budget's (OMB) Statistical Policy Directive 14 (OMB, 1978). Results from the CPS ASEC are discussed in the annual report *Income, Poverty, and Health Insurance Coverage in the United States: 2007* (DeNavas-Walt, Proctor and Smith, 2008). Some differences between the CPS ASEC and the ACS are discussed in a prior document sent to HHS (Powers, 2007). Internet links to related Census Bureau materials are also included in the Appendix of that document.

4 Results from Testing for a Rise in State Poverty Rates

We use a 10-percent significance level for our hypothesis tests. For a set of 51 tests (for the 50 states and the District of Columbia) with the standard normal *z*-statistic, the Bonferroni one-tailed critical value is 2.88. If any state has a 1.05 *z*-statistic greater than or equal to 2.88, then there is evidence that the child poverty rate for that state increased by 5 percent or greater. We find that no state has a 1.05 *z*-statistic greater than or equal to 2.88 when comparing 2006 and 2007 child poverty rate estimates. Thus, using the Bonferroni test, *we do not find statistical evidence that any state has a child poverty rate increase of 5 percent or greater between 2006 and 2007.*

As described, the Bonferroni approach is appropriate for answering the question "Is there evidence that *any* state had a child poverty rate increase of 5 percent or greater?" A different critical value would be appropriate to test for evidence of a child poverty rate increase of 5 percent or greater in a *particular* state that was selected in advance, i.e., not selected based on looking at the results for all the states. The critical value when an individual state is selected in advance is 1.28, the cutoff for a single one-tailed test with a 10 percent significance level.

No state had a 1.05 z-statistic greater than or equal to 1.28. Therefore, even ignoring multiple comparison issues and running separate 10-percent tests for each state individually, no states show a statistically significant increase in the child poverty rate of 5 percent or greater between 2006 and 2007.

Data for each state are presented in *Tables 1* and *2*. Table 1 contains poverty rate estimates, and Table 2 contains standard errors and *z*-statistics. The critical value of 2.88 should be used when checking for statistically significant evidence (at the 10-percent level) that any state had a child poverty rate increase, or an increase of 5 percent or greater. The critical value of 1.28 should be used by individual states examining their own results separately.

5 Mathematical Details

5.1 State Poverty Models

The SAIPE program's poverty models employ both direct survey-based estimates of poverty from the ACS and regression predictions of poverty based on administrative records, intercensal population estimates and Census 2000 data. The state poverty models are defined as follows:

$$\begin{aligned} y_i &= Y_i + e_i \qquad e_i \sim N(0, V_{ei}) \\ Y_i &= X_i \beta_i + u_i \quad u_i \sim N(0, \sigma_{ui}^2 I) \end{aligned},$$

where

y_i = vector of 51 state ACS direct estimates of poverty ratios for a given age group and a given year,

Y_i = vector of "true" poverty ratios for a given age group and a given year,

X_i = matrix of predictor variables for a given age group and a given year; β_i is the corresponding vector of regression coefficients,

u_i = vector of model errors for a given age group and a given year, assumed independent across states; σ^2_{ui} is their common variance,

e_i = vector of sampling errors for a given age group and a given year, assumed independent across states; V_{ei} is the diagonal matrix of the sampling error variances for each state for a given age group and a given year.

Poverty ratios for children ages 0-4 and poverty ratios for children ages 5-17 are modeled separately. The subscript i = 1, 2, 3, 4 indexes the four ACS equations for the two years (2006 and 2007) and the two age groups (0-4 and 5-17) according to the following scheme:

$i = 1$: y_1 = 2006 ACS poverty ratio estimate for children ages 0-4
$i = 2$: y_2 = 2006 ACS poverty ratio estimate for children ages 5-17
$i = 3$: y_3 = 2007 ACS poverty ratio estimate for children ages 0-4
$i = 4$: y_4 = 2007 ACS poverty ratio estimate for children ages 5-17

The coefficient vector, β_i, and the model error variance, σ^2_{ui}, are estimated by Bayesian techniques, treating the estimated sampling error variances, V_{ei}, as known. (Estimation of σ^2_{ui} and V_{ei} is discussed in *Section 5.4*.) The Bayesian techniques combine the regression predictions with the ACS direct estimates, weighting the contribution of these two components on the basis of their relative precision, in order to obtain model-based estimates of child poverty ratios by state.

Additional documentation of the SAIPE program's poverty models and estimation procedures is available online at http://www.census.gov/did/www/saipe/methods/index.html.

5.2 Poverty Rates, Ratios, and Universes

The poverty rate is defined as the number of people in poverty (numerator) divided by the number of people in the poverty universe (denominator). The poverty universe is the persons for whom the Census Bureau can determine poverty status (either "in poverty" or "not in poverty"). The ACS poverty universe excludes (a) children ages 0 to 14 who are not related to the householder by birth, marriage, or adoption and (b) people living in military barracks, dormitories, or institutional group quarters. Due to these exclusions, state poverty universe estimates are slightly lower than the corresponding state population estimates. For discussion of poverty measurement and ACS definitions, see http://www.census.gov/hhes/www/poverty/definitions.html.

In fitting the SAIPE program's models, the concept of the poverty *ratio* is used, rather than the poverty *rate*. The poverty ratio has the same numerator as the poverty rate but uses a specific population for its denominator rather than the poverty universe. In particular, the SAIPE program uses a special population for the denominator of the poverty ratio that is different from the ACS poverty universe in that it does not exclude unrelated children ages 0 to 14, but is similar to the ACS poverty universe in that it excludes people living in military barracks, dormitories, or institutional group quarters.

The motivation for modeling poverty ratios instead of poverty rates is so that only demographic estimates of the population are needed to compute the number of people in poverty after modeling has produced the estimates of the percentages. This will be discussed further below. To compute the poverty ratios to be used in the model fitting, we use ACS direct survey estimates in both the numerators and denominators (as opposed to using demographic estimates of the population in the denominators) because the positive correlation between these ACS estimates reduces the variances of the resulting poverty ratio estimates.

After model-fitting and shrinkage estimation, we convert the resulting model-based estimates of poverty ratios for children ages 0-4 and children ages 5-17 into estimates of poverty rates for children under age 18 through the following steps:

1. Multiply the model-based estimates of poverty *ratios* for each combination (i) of age group and year by the corresponding demographic estimates of the population in order to obtain estimates of the number of children ages 0-4 and 5-17 in poverty in each state. The demographic estimates of the population are from the Census Bureau's Population Estimates Program, and these estimates are adjusted to represent the population covered by the ACS.
2. Multiply the estimated number in poverty in each state by a raking factor (defined in Section 5.3) for each combination (i) of age group and year so that the resulting state-level estimated numbers in poverty sum to the ACS national estimate for that combination of age group and year.
3. For each state add the raked estimate of the number of children ages 0-4 in poverty to the raked estimate of the number of children ages 5-17 in poverty to obtain the raked estimate of the number of children under age 18 in poverty for a given year.
4. Form the estimated poverty *rates* for children under age 18 by dividing the raked estimate of the number of children under age 18 in poverty by the demographic estimate of the poverty universe for children under age 18 (children ages 0-4 plus children ages 5-17).

Note that in the first step we multiply the estimated poverty ratios by the demographic estimates of population rather than by the survey-weighted ACS estimates of population. The demographic estimates of population have no sampling error and can be more accurate than survey-based population estimates. Thus, while ACS survey-weighted population estimates are suitable denominators for the modeled poverty ratios (due to their correlation with the poverty ratio numerators, as noted above), demographic estimates of the population are more appropriate for multiplying the model-based poverty ratio estimates to obtain the estimated numbers of children in poverty.

Since the 2006 and 2007 ACS survey data have population control reference dates of July 1, 2006 and July 1, 2007, respectively, the relevant time references for the population and poverty universe estimates used as denominators for the SAIPE poverty ratios and rates are, likewise, July 1, 2006 and July 1, 2007.

For further discussion of the denominators used in poverty rates, see http://www.census.gov/did/www/saipe/data/model/info/denominators.html.

5.3 Change Variance Estimates

This section describes mathematical details behind the computation of change variance estimates and 1.05 change variance estimates. The square roots of these variance estimates form the denominators of the *z*-statistics and 1.05 *z*-statistics used to assess possible changes in the state child poverty rates.

We represent the state demographic estimates of the population in mathematical notation as:

$N_{1k} =$ 2006 demographic estimate of population for children ages 0-4 in state k,
$N_{2k} =$ 2006 demographic estimate of population for children ages 5-17 in state k,
$N_{3k} =$ 2007 demographic estimate of population for children ages 0-4 in state k,
$N_{4k} =$ 2007 demographic estimate of population for children ages 5-17 in state k,

and we represent the state demographic estimates of the poverty universe as:

$U_{1k} =$ 2006 demographic estimate of poverty universe for children ages 0-4 in state k,
$U_{2k} =$ 2006 demographic estimate of poverty universe for children ages 5-17 in state k,
$U_{3k} =$ 2007 demographic estimate of poverty universe for children ages 0-4 in state k,
$U_{4k} =$ 2007 demographic estimate of poverty universe for children ages 5-17 in state k.

We define *scaling factors* for the two age groups in each year as:

$$r_{1k} = \frac{N_{1k}}{U_{1k} + U_{2k}}, \quad r_{2k} = \frac{N_{2k}}{U_{1k} + U_{2k}}$$
$$r_{3k} = \frac{N_{3k}}{U_{3k} + U_{4k}}, \quad r_{4k} = \frac{N_{4k}}{U_{3k} + U_{4k}},$$

and we define *raking factors* for each combination (i) of age group and year as:

$$RF_i = \frac{\text{ACS direct national estimate of number in poverty for age group and year combination } i}{\sum_k (\text{model-based estimate of number in poverty for state } k \text{ for age group and year combination } i)}.$$

The scaling factors weight the ages 0-4 and ages 5-17 estimated poverty ratios in proportion to the number of children in each age group, forming contributions to the under-age-18 poverty rates. The raking factors scale the poverty ratio estimates so that the sum of the products of poverty ratios and demographic estimates of the population equals the national ACS estimate of the number of children in poverty in each age group and year.

Letting R_i be a 51x51 diagonal matrix with the r_{ik} terms (scaling factors) on the diagonal, the vector of contributions to the 2006 under-age-18 poverty rate from the ages 0-4 group and the ages 5-17 group are $R_1 \cdot Y_1$ and $R_2 \cdot Y_2$, respectively. The raked estimators of these products are $R_1 \cdot RF_1 \hat{Y}_1$ and $R_2 \cdot RF_2 \hat{Y}_2$. Likewise, the vector of contributions to the 2007 under-age-18 poverty rate from the ages 0-4 group and the ages 5-17 group can be written as $R_3 \cdot Y_3$ and $R_4 \cdot Y_4$, respectively, and the raked estimators of these products are $R_3 \cdot RF_3 \hat{Y}_3$ and $R_4 \cdot RF_4 \hat{Y}_4$.

The error in the change estimate can then be written as:

$$[R_3(Y_3 - RF_3\hat{Y}_3) + R_4(Y_4 - RF_4\hat{Y}_4)] - [R_1(Y_1 - RF_1\hat{Y}_1) + R_2(Y_2 - RF_2\hat{Y}_2)], \qquad (2)$$

where $Y_i - RF_i\hat{Y}_i$ is the error in the raked poverty ratio estimate for combination (i) of age group and year. The diagonal of the variance matrix of this expression will be the change variance estimates. Similarly, the error in the 1.05 change estimate can be written as:

$$[R_3(Y_3 - RF_3\hat{Y}_3) + R_4(Y_4 - RF_4\hat{Y}_4)] - 1.05[R_1(Y_1 - RF_1\hat{Y}_1) + R_2(Y_2 - RF_2\hat{Y}_2)], \qquad (3)$$

and the diagonal of the variance matrix of this expression will be the 1.05 change variance estimates.

Bell (1999) determined that the vector of prediction errors, $Y_i - RF_i\hat{Y}_i$, for combination (i) of age group and year can be expressed as:

$$Y_i - RF_i\hat{Y}_i = A_i \cdot u_i + (A_i - I) \cdot e_i + A_i X_i \beta_i \ ,$$

where

$$A_i = (1 - RF_i)I + RF_i(I - H_i)(I - M_i) \ ,$$
$$H_i = \sigma_{ui}^2 \Sigma_i^{-1}, \quad \Sigma_i = \sigma_{ui}^2 I + V_{ei}, \quad \text{and } M_i = X_i(X_i' \Sigma_i^{-1} X_i)^{-1} X_i' \Sigma_i^{-1} .$$

The term $A_i X_i \beta_i$ can be rewritten as $(1 - RF_i) \times X_i \beta_i$. This is, fundamentally, a bias term that arises from raking state estimates to national totals under the model assumption that the regression function $X_i \beta_i$ produces unbiased estimates. (The raking factor, RF_i, also includes some random estimation error.) The model is, of course, an approximation, and the raking is done because it is believed to reduce possible bias arising from failure of the model assumptions. We therefore ignore this bias term in computing measures of error for the raked estimates and compute the covariance matrix based on just the first two terms, $A_i u_i$ and $(A_i - I) \cdot e_i$.

Proceeding with the assumption that the $A_i X_i \beta_i$ term can be ignored, the errors in the change and 1.05 change estimates shown in formulas (2) and (3) can be re-written as:

$$\begin{aligned} &R_3[A_3 \cdot u_3 + (A_3 - I) \cdot e_3] + R_4[A_4 \cdot u_4 + (A_4 - I) \cdot e_4] \\ &\quad + \widetilde{R}_1[A_1 \cdot u_1 + (A_1 - I) \cdot e_1] + \widetilde{R}_2[A_2 \cdot u_2 + (A_2 - I) \cdot e_2] \end{aligned} , \qquad (4)$$

where $\widetilde{R}_1$ and $\widetilde{R}_2$ are $-1.05R_1$ and $-1.05R_2$, respectively, for the error in the 1.05 change estimate and are $-R_1$ and $-R_2$, respectively, for the error in the change estimate.

The covariance matrix of formula (4) can, then, be written as:

$$\sum_i \sum_j [\overline{R}_i \cdot (A_i - I)]Cov(e_i, e_j)[\overline{R}_j \cdot (A_j - I)]' + \sum_i \sum_j [\overline{R}_i \cdot A_i]Cov(u_i, u_j)[\overline{R}_j \cdot A_j]' \quad , \qquad (5)$$

where, for the 1.05 change variance estimates:

$\overline{R}_i = -1.05R_i$ when i = 1 or 2, and $\overline{R}_i = R_i$ when i = 3 or 4,

and, for the change variance estimates:

$\overline{R}_i = -R_i$ when i = 1 or 2, and $\overline{R}_i = R_i$ when i = 3 or 4.

In formula (5), $[Cov(e_i, e_j)]$ and $[Cov(u_i, u_j)]$ are 51x51 matrices. They are diagonal matrices (i.e., the off-diagonal elements are all zero) because we assume the sampling errors and model errors are uncorrelated across states and uncorrelated with each other.[1] The 16 different i, j pairs correspond to the individual cells in the figure below:

		1	2	3	4
		04yr1	517yr1	04yr2	517yr2
1	04yr1	v11	v12	v13	v14
2	517yr1	v21	v22	v23	v24
3	04yr2	v31	v32	v33	v34
4	517yr2	v41	v42	v43	v44

There are 32=16+16 terms altogether in formula (5)'s summation. The change variance and 1.05 change variance estimates are then the 51 diagonal elements of formula (5) evaluated.

[1] Note the off-diagonal elements of $[Cov(e_i, e_j)]$ and $[Cov(u_i, u_j)]$ involve two different states. The former are zero because the ACS samples for different states are selected independently, and the latter are assumed to be zero by the model.

5.4 Covariances Needed for Change Variance Estimates

In order to estimate formula (5), we need to estimate the diagonal elements of $[Cov(e_i, e_j)]$ and $[Cov(u_i, u_j)]$. We do this through the following steps:

- Compute direct estimates of state ACS sampling error variances;
- Compute averages over states of direct estimates of the ACS sampling error correlations between the ages 0-4 poverty ratio and the ages 5-17 poverty ratio;
- Fit models to the direct ACS state estimates to produce the state model-based poverty ratio predictions;
- Treat pairs of ACS state equations (by age group and year) jointly and use Bayesian techniques to estimate the correlation between model errors in the two equations; and
- Combine the estimated sampling error variances and correlations to obtain estimated sampling error covariances, and combine the estimated model error variances and correlations to obtain estimated model error covariances.

These steps are described in more detail below.

Sampling Error Variances

The state ACS sampling error variances, V_{ei}, for each age-group poverty ratio (0-4 and 5-17) are estimated directly using a successive difference replication method. This variance estimation method is described in Chapter 12 of *Design and Methodology, American Community Survey* (U.S. Census Bureau, 2009). The theoretical framework for this method was originally discussed by Wolter (1984) and extended by Fay and Train (1995).

Sampling Error Correlations

The sampling error correlations between the 0-4 poverty ratio and the 5-17 poverty ratio within a given year (i.e., ρ_{e12} and ρ_{e34}) are estimated by averaging the corresponding direct state estimates of sampling error correlations across states. Specifically, we use the successive difference replication method to compute direct estimates of covariances between 0-4 poverty ratio and 5-17 poverty ratio by state. We then construct the corresponding correlation estimates and average these state correlations over the 50 states and the District of Columbia. The 2006 average correlations are computed separately from the 2007 average correlations. In each case, simple unweighted averages of the correlations are used.

The sampling error correlations between the 2006 poverty ratio and the 2007 poverty ratio (i.e., ρ_{e13}, ρ_{e24}, ρ_{e14} and ρ_{e23}) are assumed to be zero, as adjacent years of single-year ACS estimates have no household sample overlap. We examined this assumption by using the successive difference replication method to compute direct estimates of covariances between the 2006 poverty ratio and the 2007 poverty ratio by state. Taking the simple average of the corresponding correlation estimates over the 50 states and the District of Columbia, we observed the estimated year-to-year correlations to be near zero, as expected.

Model Error Variance

The model error variance, σ_{ui}^2, for each age-group poverty ratio (0-4 and 5-17) is estimated while fitting the state models to the ACS direct poverty ratio estimates. We use a Bayesian approach in estimation of the state model, and we regard σ_{ui}^2 as estimated by its posterior mean. We use a noninformative (flat) prior for all model parameters.

Model Error Correlations

The model error correlations (ρ_{u12}, ρ_{u13}, ρ_{u14}, ρ_{u23}, ρ_{u24}, ρ_{u34}) are estimated using the Bayesian approach and treating each pair of ACS state equations jointly. Unlike for the sampling error correlations, we do not assume zero year-to-year model error correlations. For each of the six possible distinct pairs of equations among the four ACS state equations, we specify flat prior distributions for the regression coefficients and the model error variances, as was done when fitting the models one equation at a time. The prior for the model error correlation is taken to be uniform on the interval [-1,1]. We then take the posterior mean of the model error correlation as its point estimate. This joint Bayesian treatment of two ACS equations simultaneously is done using the WinBUGS package (Spiegelhalter et al., 2003).

Note that although this joint-equation model-fitting procedure produces new estimates of the other model parameters involved in each pair of equations (i.e., the regression parameters and model error variances), the only result used from each joint model-fitting is the estimate of the model error correlation. This is done in order to remain consistent with the results obtained from fitting the single ACS equations separately (see Section 5.1).

Covariances

Finally, in order to estimate the sampling error covariances in formula (5), we combine the estimated sampling error variances with the estimated sampling error correlations as follows:

$$Cov(e_i, e_j) = \rho_{eij}(V_{ei}V_{ej})^{1/2} \quad ,$$

where $(V_{ei}V_{ej})^{1/2}$ represents the matrix formed by taking the square root of each element of the diagonal matrix $V_{ei}V_{ej}$. Likewise, in order to estimate the model error covariances, we combine the estimated model error variances with the estimated model error correlations as follows:

$$Cov(u_i, u_j) = \rho_{uij}\sigma_{ui}\sigma_{uj}I \quad .$$

There are ten distinct sampling error covariances for each state, of which four are sampling error variances, and there are ten distinct model error covariances (all common to the 50 states and the District of Columbia), of which four are model error variances. These covariances are together used to evaluate formula (5) at the end of Section 5.3.

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Table 1. Poverty Estimates for Testing for a Rise in State Child Poverty Rates of Five Percent or Greater: 2006 to 2007

FIPS code	name		under age 18 poverty rate 2006	under age 18 poverty rate 2007	% change poverty rate[1] '06-'07	change estimate[2] '06-'07	1.05 change estimate[3] '06-'07
01	Alabama		23.1	23.6	2.2	0.5	-0.7
02	Alaska		14.6	13.1	-10.3	-1.5	-2.2
04	Arizona		19.8	20.0	1.0	0.2	-0.8
05	Arkansas		24.6	25.3	2.8	0.7	-0.5
06	California		18.1	17.3	-4.4	-0.8	-1.7
08	Colorado		14.8	15.3	3.4	0.5	-0.2
09	Connecticut		10.9	10.8	-0.9	-0.1	-0.6
10	Delaware		14.8	14.5	-2.0	-0.3	-1.0
11	District of Columbia		28.3	25.7	-9.2	-2.6	-4.0
12	Florida		17.6	17.3	-1.7	-0.3	-1.2
13	Georgia		20.3	19.8	-2.5	-0.5	-1.5
15	Hawaii		11.8	10.8	-8.5	-1.0	-1.6
16	Idaho		16.0	15.9	-0.6	-0.1	-0.9
17	Illinois		17.0	16.6	-2.4	-0.4	-1.3
18	Indiana		17.4	17.1	-1.7	-0.3	-1.2
19	Iowa		13.8	13.7	-0.7	-0.1	-0.8
20	Kansas		15.4	14.7	-4.6	-0.7	-1.5
21	Kentucky		23.1	23.6	2.2	0.5	-0.7
22	Louisiana		28.2	26.9	-4.6	-1.3	-2.7
23	Maine		16.9	15.7	-7.1	-1.2	-2.1
24	Maryland		10.1	10.6	5.0	0.5	0.0
25	Massachusetts		12.6	13.0	3.2	0.4	-0.2
26	Michigan		18.3	19.3	5.5	1.0	0.1
27	Minnesota		12.0	11.9	-0.8	-0.1	-0.7
28	Mississippi		29.5	29.4	-0.3	-0.1	-1.6
29	Missouri		19.3	18.4	-4.7	-0.9	-1.9
30	Montana		19.3	18.9	-2.1	-0.4	-1.4
31	Nebraska		14.4	14.7	2.1	0.3	-0.4
32	Nevada		14.3	14.9	4.2	0.6	-0.1
33	New Hampshire		9.6	9.2	-4.2	-0.4	-0.9
34	New Jersey		11.7	11.4	-2.6	-0.3	-0.9
35	New Mexico		25.6	25.2	-1.6	-0.4	-1.7
36	New York		20.1	19.6	-2.5	-0.5	-1.5
37	North Carolina		20.1	19.5	-3.0	-0.6	-1.6
38	North Dakota		14.0	14.0	0.0	0.0	-0.7
39	Ohio		18.5	18.4	-0.5	-0.1	-1.0
40	Oklahoma		23.5	22.2	-5.5	-1.3	-2.5
41	Oregon		17.6	17.2	-2.3	-0.4	-1.3
42	Pennsylvania		16.7	16.2	-3.0	-0.5	-1.3
44	Rhode Island		16.0	16.9	5.6	0.9	0.1
45	South Carolina		22.2	21.1	-5.0	-1.1	-2.2
46	South Dakota		17.7	17.5	-1.1	-0.2	-1.1
47	Tennessee		22.6	22.5	-0.4	-0.1	-1.2
48	Texas		23.8	23.1	-2.9	-0.7	-1.9
49	Utah		12.3	11.3	-8.1	-1.0	-1.6
50	Vermont		12.6	12.4	-1.6	-0.2	-0.8
51	Virginia		12.3	12.9	4.9	0.6	0.0
53	Washington		15.5	15.0	-3.2	-0.5	-1.3
54	West Virginia		24.9	23.4	-6.0	-1.5	-2.8
55	Wisconsin		14.7	14.5	-1.4	-0.2	-0.9
56	Wyoming		13.6	12.8	-5.9	-0.8	-1.5

[1] 100 × [(2007 Poverty Rate Estimate – 2006 Poverty Rate Estimate)/(2006 Poverty Rate Estimate)]
Percent changes may not be statistically significant even if they appear greater than five percent. See Section 4 for details.

[2] 2007 Poverty Rate Estimate – 2006 Poverty Rate Estimate

[3] 2007 Poverty Rate Estimate – 1.05 × (2006 Poverty Rate Estimate)

Source: Small Area Income and Poverty Estimates (SAIPE) program, U.S. Census Bureau

Table 2. Standard Errors and z-statistics for Testing for a Rise in State Child Poverty Rates of Five Percent or Greater: 2006 to 2007

FIPS code	name	change estimate[1] '06-'07	S.E. of change est. '06-'07	z-statistic[2] '06-'07	1.05 change estimate[3] '06-'07	S.E. of 1.05 change est. '06-'07	1.05 z-statistic[4] '06-'07
01	Alabama	0.5	0.58	0.92	-0.7	0.59	-1.05
02	Alaska	-1.5	0.62	-2.38	-2.2	0.64	-3.45
04	Arizona	0.2	0.60	0.35	-0.8	0.61	-1.28
05	Arkansas	0.7	0.63	1.02	-0.5	0.65	-0.91
06	California	-0.8	0.30	-2.40	-1.7	0.31	-5.25
08	Colorado	0.5	0.55	0.90	-0.2	0.56	-0.44
09	Connecticut	-0.1	0.56	-0.16	-0.6	0.58	-1.10
10	Delaware	-0.3	0.62	-0.58	-1.0	0.64	-1.73
11	District of Columbia	-2.6	1.21	-2.14	-4.0	1.23	-3.25
12	Florida	-0.3	0.42	-0.82	-1.2	0.43	-2.86
13	Georgia	-0.5	0.47	-1.03	-1.5	0.48	-3.11
15	Hawaii	-1.0	0.74	-1.29	-1.6	0.76	-2.04
16	Idaho	-0.1	0.64	-0.13	-0.9	0.66	-1.34
17	Illinois	-0.4	0.43	-0.97	-1.3	0.44	-2.88
18	Indiana	-0.3	0.52	-0.51	-1.2	0.53	-2.13
19	Iowa	-0.1	0.56	-0.27	-0.8	0.57	-1.49
20	Kansas	-0.7	0.57	-1.16	-1.5	0.58	-2.45
21	Kentucky	0.5	0.58	0.84	-0.7	0.60	-1.11
22	Louisiana	-1.3	0.65	-1.97	-2.7	0.67	-4.01
23	Maine	-1.2	0.68	-1.71	-2.1	0.70	-2.88
24	Maryland	0.5	0.51	1.00	0.0	0.53	0.02
25	Massachusetts	0.4	0.49	0.68	-0.2	0.50	-0.60
26	Michigan	1.0	0.44	2.18	0.1	0.46	0.12
27	Minnesota	-0.1	0.46	-0.14	-0.7	0.47	-1.43
28	Mississippi	-0.1	0.73	-0.17	-1.6	0.74	-2.15
29	Missouri	-0.9	0.55	-1.55	-1.9	0.56	-3.24
30	Montana	-0.4	0.68	-0.51	-1.4	0.70	-1.88
31	Nebraska	0.3	0.60	0.50	-0.4	0.61	-0.69
32	Nevada	0.6	0.63	1.01	-0.1	0.65	-0.12
33	New Hampshire	-0.4	0.66	-0.62	-0.9	0.67	-1.32
34	New Jersey	-0.3	0.44	-0.54	-0.9	0.45	-1.83
35	New Mexico	-0.4	0.68	-0.50	-1.7	0.70	-2.32
36	New York	-0.5	0.36	-1.45	-1.5	0.37	-4.11
37	North Carolina	-0.6	0.48	-1.13	-1.6	0.50	-3.12
38	North Dakota	0.0	0.68	-0.05	-0.7	0.69	-1.07
39	Ohio	-0.1	0.46	-0.19	-1.0	0.48	-2.13
40	Oklahoma	-1.3	0.58	-2.24	-2.5	0.60	-4.15
41	Oregon	-0.4	0.63	-0.56	-1.3	0.65	-1.90
42	Pennsylvania	-0.5	0.43	-1.05	-1.3	0.44	-2.93
44	Rhode Island	0.9	0.68	1.37	0.1	0.69	0.18
45	South Carolina	-1.1	0.55	-1.85	-2.2	0.57	-3.76
46	South Dakota	-0.2	0.73	-0.28	-1.1	0.74	-1.46
47	Tennessee	-0.1	0.59	-0.24	-1.2	0.60	-2.12
48	Texas	-0.7	0.35	-1.85	-1.9	0.36	-5.16
49	Utah	-1.0	0.56	-1.72	-1.6	0.57	-2.75
50	Vermont	-0.2	0.69	-0.42	-0.8	0.70	-1.32
51	Virginia	0.6	0.46	1.30	0.0	0.47	-0.03
53	Washington	-0.5	0.48	-1.21	-1.3	0.49	-2.77
54	West Virginia	-1.5	0.66	-2.34	-2.8	0.68	-4.12
55	Wisconsin	-0.2	0.50	-0.39	-0.9	0.51	-1.82
56	Wyoming	-0.8	0.72	-1.13	-1.5	0.73	-2.04

[1] 2007 Poverty Rate Estimate – 2006 Poverty Rate Estimate

[2] $((2007\ \text{Poverty Rate}) - (2006\ \text{Poverty Rate}))/\sqrt{\text{Var}((2007\ \text{Poverty Rate}) - (2006\ \text{Poverty Rate}))}$

[3] 2007 Poverty Rate Estimate – 1.05 × (2006 Poverty Rate Estimate)

[4] $((2007\ \text{Poverty Rate}) - 1.05\times(2006\ \text{Poverty Rate}))/\sqrt{\text{Var}((2007\ \text{Poverty Rate}) - 1.05\times(2006\ \text{Poverty Rate}))}$

See Section 4 for discussion of critical values. *Source:* SAIPE program, U.S. Census Bureau