

Investigating Quarterly Trading Day Effects

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Abstract

When the U. S. Census Bureau began looking into seasonal adjustment of the Quarterly Services Survey series, one question was whether trading day adjustments were reasonable. The series started in the fourth quarter of 2003, and consideration of trading day adjustments began when series had only 17 quarters of data. Beyond concern over series length, however, we questioned more generally whether trading day effects are measurable in quarterly series. To answer our questions, we started with monthly Census Bureau series known to have significant trading day effects. We summed to quarterly levels and tested for significance. We tested using both real and simulated effects. This paper discusses the results of our research.

Key words: seasonal adjustment, simulated data, time series

Disclaimer

This report is released to inform interested parties of ongoing research and to encourage discussion of work in progress. Any views expressed on statistical or methodological issues are those of the authors and not necessarily those of the U.S. Census Bureau.

1. Introduction

The U.S. Census Bureau publishes thousands of seasonally adjusted time series each month. Part of the seasonal adjustment procedure involves adjusting for the weekday composition of the month, called the trading day effect. For example, given the same economic climate and consistency in other effects, building-permit authorizations measured in August 2007 when there were five Wednesdays, Thursdays, and Fridays, may be expected to have been higher than in August 2008 when there were five Fridays, Saturdays, and Sundays. Building permit offices in the United States usually are open on weekdays (Monday through Friday) and closed on weekends (Saturday and Sunday).

X-13A-S (X-13-ARIMA-SEATS) (U. S. Census Bureau 2009), the Census Bureau's seasonal adjustment software that follows in the line of X-11 (Shiskin, Young, and Musgrave 1967), X-12-ARIMA (Findley, Monsell, Bell, Otto, and Chen 1998), and Statistics Canada's X-11-ARIMA and updates (Dagum 1988) have built-in regression variables to estimate trading day effects as well as statistical tests to help users determine if the effects are present.

Trading day effects in monthly data often are quite evident. Aside from nonleap-year Februaries that have exactly four weeks, all other months have some difference in day-of-week composition; that is, some days of the week occur five times. Months in a year may be the same: in leap years, January and July have the same composition, but most months differ from each other. With sufficiently long time series, there are enough data to

fit a regression model to estimate the effects. Generally seven years is considered a sufficient length for estimating trading day effects in monthly time series (Statistics New Zealand).

By contrast, the trading day effects in quarterly time series can be subtle. Quarters are 90, 91, or 92 days long. The second quarter always is 91 days long or exactly 13 weeks, so there is no trading day effect present in that quarter. During leap years, the first quarter also is 91 days or exactly 13 weeks. Nonleap-year first quarters are 90 days long, short of 13 weeks by one day. Third and fourth quarters are 92 days long, one day longer than 13 weeks. In monthly series, each day of the week occurs four or five times; In quarterly series, days of the week occur 12, 13, or 14 times.

In addition to quarters being equal or close to 13 weeks, their composition tends to be similar. Within the same year, the extra day for the third quarter is the day previous to the extra day for the fourth quarter. That is, if the day of the week that occurs 14 times in the third quarter is a Wednesday, then the day of the week that occurs 14 times in the fourth quarter is a Thursday. If these days of the week have a similar effect, then the third and fourth quarters will have similar trading day effects. There is variation, however. The day of the week that occurs only 12 times in a nonleap-year first quarter is the same day of the week that occurs 14 times in the third quarter. If that day of the week carries a lot of importance in the trading day pattern, then these quarters might demonstrate the biggest difference in trading day effects. Overall, the similarity in quarters may make it difficult to estimate effects if they are not especially strong and consistent.

The Census Bureau publishes seasonally adjusted quarterly time series but has not adjusted any of these series for trading day effects. Recently the Census Bureau reviewed the Quarterly Services Survey (QSS) data to determine whether the series could be seasonally adjusted. The series are quite short; data collection began in the fourth quarter of 2003, but for many series a seasonal pattern was apparent. At least some of the QSS series reasonably could be expected to have trading day effects as they are measures of services that occur with different frequency on different days of the week.

To examine this expectation we devised a practical test of quarterly trading day effects by generating quarterly series that we expected to have trading day effects and then testing whether those effects are identifiable. Starting with monthly time series that clearly demonstrated significant trading day effects and summing the series to a quarterly basis, we checked how often the trading day effects were preferred. We expected that the quarterly series would have evidence of trading day effects just as the monthly series did.

2. Background

The following descriptions explain the regressors, diagnostics, and general methods we used. These regressors are readily available from X-13A-S, and each run generates these diagnostics.

The current seasonal adjustment method at the Census Bureau uses modeling for three primary purposes, (1) to extend the series with forecasts to improve the performance of the centered seasonal moving averages, (2) to estimate outlier effects and replace those before estimating the series components, and (3) to directly estimate regression effects like trading day and moving holidays (Easter, for example).

The models consist of Autoregressive Integrated Moving Average (ARIMA) models combined with regression effects. These are called regARIMA models.

The most common trading day regression involves six regressors:

$$TD_{1t} = (\# \text{ of Mondays}) - (\# \text{ of Sundays})$$

$$TD_{2t} = (\# \text{ of Tuesdays}) - (\# \text{ of Sundays})$$

...

$$TD_{6t} = (\# \text{ of Saturdays}) - (\# \text{ of Sundays})$$

where t is the month or quarter. The overall effect is constrained to zero; the Sunday regressor is derived from the sum of the other days' effects (Bell and Hillmer 1983, U. S. Census Bureau 2009).

For comparisons, we used Akaike's Information Criterion (AIC) (Akaike 1973) corrected for sample size. The modified diagnostic is called AIC_C (Hurvich and Tsai 1989). It is a comparison measure, and minimum values are preferred (U. S. Census Bureau 2009). The original AIC diagnostic is defined as

$$AIC = -2L_N + 2p$$

where L is the maximized value of the log-likelihood function and p is the number of estimated parameters. The AIC_C is

$$AIC_C = -2L + \frac{2p}{1 - \frac{p+1}{N}}$$

where L and p are as above and N is the number of observations (model span length). As N increases, the AIC_C becomes closer to the AIC. To use the AIC or AIC_C diagnostic, certain model features should be the same in the models being compared. Of particular note for our study, the outliers should be the same (U. S. Census Bureau 2009).

Some of the series we investigated had nested models. In other words, fixing model coefficients to zero, in this case setting the coefficients for the trading day regressors to zero, results in the other model, in this case the model without trading day effects. We used the AIC_C diagnostic regardless of whether the models were nested because this approach is consistent with the automatic model identification method and because we wanted to be consistent in our comparisons. In addition to AIC_C comparisons, however, we also looked at significance of the trading day effects as measured by chi square statistics.

Spectral diagnostics are available for checking whether trading day effects are present or have been effectively removed from the series during adjustment. For quarterly series, X-13A-S does not display trading day frequencies in the spectral graphs unless the model span is at least 15 years long (60 observations). A sufficiently large peak occurring at a trading day frequency in the graph indicates the presence of trading day effects (Soukup and Findley 1999). Ladiray (2008) has identified new frequencies for use with the spectral diagnostic when working with quarterly series. His results indicated that the current frequencies available from X-13A-S are not reliable for identifying quarterly trading day effects. We did not use spectral diagnostics to evaluate our results.

3. Two Approaches

We took two approaches to investigating quarterly trading day effects. The initial approach involved real data, and the second used simulated series based on the real series. Our data series started in 1992 and ended in 2007, giving us 16 years of data, or 64 quarters.

For these years, it is worthwhile to check the distribution of the days of the week for the quarters. Except for leap years, the first quarter is short of 13 weeks by one day. For our series the first quarter was short each day at least once. The third and fourth quarters are one day more than 13 weeks, and for our series each day was the extra day at least four times. Given that each day of the week was represented in each circumstance, it seemed that sufficient data were available for modeling the regression. Table 1 shows more information about the frequencies.

	First (Short)	Third (Extra)	Fourth (Extra)
Monday	1	2	2
Tuesday	2	2	2
Wednesday	1	2	2
Thursday	2	3	2
Friday	2	2	3
Saturday	2	3	2
Sunday	2	2	3

For both sets of data we used X-13A-S Build 139, compiled April 1, 2009, for our final model results (U. S. Census Bureau 2009). In addition, we used Win X-12, a Windows interface to X-12-ARIMA and X-13A-S (Lytras 2008), and X-12-Data, an Excel macro program that converts files from Excel to text formats compatible with X-12-ARIMA and X-13A-S or from text to Excel (Feldpausch with updates 2009).

4. Real Series

4.1 Real Series: Methods

For our investigation with real data, we started with monthly time series with strong trading day effects, and we summed the data by quarters to see if the effects seen in the monthly series would be evident in the quarterly series.

Our data were 70 Census Bureau monthly series: 12 Building Permit Authorizations series (www.census.gov/const/www/newresconstindex_excel.html) and 58 Retail Trade and Food Services series (www.census.gov/mrts/www/mrts.html). Descriptions of data sources and reliability are available from web sites www.census.gov/const/www/newresconstdoc.html and www.census.gov/retail/mrts/how_surveys_are_collected.html. We chose to use only some of the 67 published Retail series. We eliminated series whose data are available only back to 2001; department store series that include leased departments within the stores, as these are not included in the published aggregates; and a series whose sample changed late in the series, changing the established seasonal pattern, as noted on the web site. In addition, we eliminated one Retail series for which automatic testing indicated no significant trading

day effects in the monthly data. Some of the series were aggregates of others. For example, Northeast Total Building Permits includes Northeast Single Family Building Permits. Because of this aggregation, the results of the series were interrelated, but we kept the series because we wondered if amount of aggregation would affect how often a trading day effect was identified. Table 2 shows the breakout of aggregation of the series. The least-aggregated level was the lowest level of publication. We considered a series to be more aggregated or in the middle level of aggregation if it included at least one of the least aggregated published series as a subcategory. But this more-aggregated level was different from the highest level of aggregation because we considered each of them still to be one definable category. The most aggregated series were high-level sums of various categories.

The least aggregated Building Permits series were Single Family Construction by region (four series), 2–4 Unit Residential Construction, and 5-or-More Unit Residential Construction. The least aggregated Retail series were mostly five-digit NAICS series; some were four-digit or six-digit NAICS series, and one was a three-digit series.

The more aggregated Building Permits series were Total Residential Construction by region. The more aggregated Retail series were mostly three-digit NAICS series, some four-digit NAICS series, one five-digit NAICS series, a sum of two three-digit NAICS series, and a sum of two four-digit NAICS series.

The most aggregated Building Permits series were U. S. Single Family Construction and U. S. Total; the most aggregated Retail series were Total Retail and Food Services Sales, Total Retail Sales and Food Services Excluding Motor Vehicle and Parts, Total Retail Sales, Total Retail Sales excluding Motor Vehicle and Parts Dealers, and one series totaling Retail Sales for NAICS codes 442, 443, 448, 451, 452, and 4532.

Table 2: Number of Series by Level of Aggregation				
	Aggregation Level			
	Least	More	Most	Total
Building Permits	6	4	2	12
Retail Trade	31	22	5	58
Total	37	26	7	70

Preliminary runs with the monthly series showed that the log transformation (multiplicative adjustment) was preferred for more than half (7 of 12) of the Building Permit series and for 91% (53 of 58) of the Retail series. Because of the more-intuitive interpretation of results and diagnostics from multiplicative adjustments and to keep this aspect consistent, we chose to use the log transformation for all of the series.

Next, we summed the monthly series to quarters and ran automatic model identification, allowing ARIMA models of maximum nonseasonal order two and seasonal order one (default settings) and automatic outlier identification of additive outliers (point outliers), level shifts, and temporary changes. For the Retail series, we also allowed identification of Easter effects. For both sets of series we automatically identified trading day effects. If trading day effects were identified in this early step, we considered them significant through all following steps.

Based on prior knowledge of the series, we also modeled specific outliers and used a lowered critical value of 2.00 to judge their significance; the default critical value was 3.703. The Building Permits series had changes in sample, essentially increases in the sample universe, in January 1994 and January 2004. We specified level shifts for the first quarter of 1994 and 2004. Because the monthly Retail series were affected by the September 11, 2001 terrorist attacks in the U. S., we specified an additive outlier for the third quarter of 2001. After estimating the model, we retained the outlier regressors only if they were significant by our lowered critical value. We retained the regressors in Building Permits series if the t statistics were greater than 2.00, and in the Retail series if the t statistics were less than -2.00 . Four of the Building Permits series retained a level shift at the first quarter of 1994, but none retained the 2004 level shift. Nine of the Retail series retained the additive outlier regressor.

Under automatic modeling, X-12-ARIMA uses a default model when performing the tests for presence of trading day and Easter effects. Because of the potential model differences and because of differences in model parameter estimation, AIC_c results can differ slightly when using automatic modeling and when model choices are set. For the final models, we completed runs for comparisons by incorporating trading day regressors into all models. This step allowed us to see if the effects were close to being significant when they were not chosen. When estimating the new models with trading day effects, some additional outliers were identified. We incorporated those additional outliers into both the models with and without the trading day regressors so that the outlier sets would be identical, allowing us to compare AIC_c values.

4.2 Real Series: Results

In our initial run with quarterly series of real data X-13A-S automatically identified trading day effects for only 23% (16 of 70) of the series. When we fit trading day regressors for the remaining series' models to compare AIC_c results, we saw newly-identified outliers for 19% (10 of 54) of those series, and we fit regressors for those outliers in the final models.

Because we expected these series to have trading day effects, we provided additional chances to identify the effects after the initial automatic modeling. With the final models, an additional six series had AIC_c results that indicated the presence of trading day effects. Four of the 10 series with newly-identified outliers now indicated that there were trading day effects. Table 3 shows how many series had AIC_c results, either during automatic modeling or during the estimation with the final model, that favored using trading day effects, broken out by level of aggregation. The number is given as well as the percentage of series of that series type and aggregation level (as described above, numbers shown in Table 2). We do not have enough series to decide on the significance of these results, so we cannot say if high levels of aggregation improve the chance of correctly identifying trading day effects, but this possibility is interesting and worth further investigation. The fact that only one Building Permits series had apparent trading day effects was a surprise.

The regression effects estimated for the quarterly series were not always closely related to those estimated for the monthly series. Table 4 shows correlations between the monthly and quarterly estimated regression coefficients. Of particular interest are the Tuesday and Thursday coefficients where the correlations are negative and the Wednesday correlation that is quite small. The estimated effects are not especially similar for those days. For the series whose quarterly effects were not identified, we did not expect to see a strong similarity to the monthly effects.

Of the 48 series whose AIC_C results did not indicate trading day effects were present, the AIC_C differences ranged from -0.01 to -20.64. The high number of series with large AIC_C differences indicates that the effects were not strong in these quarterly series. Table 5 shows the distribution of these differences.

Table 3: Series With AIC_C Favoring Use of Trading Day Effects by Level of Aggregation								
	Aggregation Level							
	Least		More		Most		Total	
Building Permits	17%	1 of 6	0%	0 of 4	0%	0 of 2	8%	1 of 12
Retail Sales	32%	10 of 31	36%	8 of 22	60%	3 of 5	36%	21 of 58
Total	30%	11 of 37	31%	8 of 26	43%	3 of 7	31%	22 of 70

Table 4: Correlations of Monthly and Quarterly Estimated Trading Day Regression Coefficients		
	Quarterly Effects Identified	Quarterly Effects Not Identified
Monday	0.64	-0.16
Tuesday	-0.29	0.48
Wednesday	0.06	0.07
Thursday	-0.43	0.24
Friday	0.93	0.32
Saturday	0.86	0.30
Sunday	0.49	0.56

The maximum p value for the chi square statistic for the trading day effects of the monthly series was 0.01 for one of the 70 series; for the other monthly series the p value was 0.00. For the quarterly series, the p value was as high as 0.29 for the series for which AIC_C indicated trading day effects were present but for the series for which AIC_C did not identify trading day effects, the maximum p value was 0.97, and the p value was above 0.50 for 19% (9 of 48) of the series. As with the AIC_C results, the evidence indicates that just because the effects were obvious in the original monthly series, they are not necessarily identifiable in the quarterly series.

Table 5: Range of AIC_C Differences for Quarterly Series With No Identified Trading Day Effects		
AIC_C Difference	Number of Series	
$-2 \leq \text{Diff} < 0$	8%	4
$-4 \leq \text{Diff} < -2$	10%	5
$-6 \leq \text{Diff} < -4$	27%	13
$-8 \leq \text{Diff} < -6$	13%	6
$\text{Diff} < -8$	42%	20
Total	48	

5. Simulated Series: Methods

5.1 Simulated Series: Methods

We generated simulated series using components from the monthly Building Permits and Retail Sales series as a base. We seasonally adjusted the series and then recombined the seasonal and trend components into new series. We did not use the irregular components. This type of data simulation is described in Hood, Ashley, and Findley (2000). As with their simulations, we used three different trend components: one very smooth, uncomplicated trend with few changes in direction (Figure 1), one somewhat more complex (Figure 2), and a third even more complex (Figure 3).

We selected three ranges of seasonal factors: small (0.90–1.07), medium (0.70–1.23), and large (0.63–3.16). We multiplied each trend component by each seasonal component to create nine base series. We then imposed two sets of trading day effects on the base series: (a) a set of coefficients from a series that AIC_C indicated as having trading day effects at the quarterly level (we will call these the “found” effects) and (b) coefficients from a series whose effects were not identified at the quarterly level and whose p value was 0.77 for the regression chi square statistic (the “not-found” effects). We generated the effects by running the trading day regression with fixed coefficients, saving the trading day factors, then multiplying each set of factors by each base series. We also tested the base series without any imposed trading day effects. These combinations resulted in 27 series. Our hope was that without the irregular component, the trading day effects would be more easily identifiable.

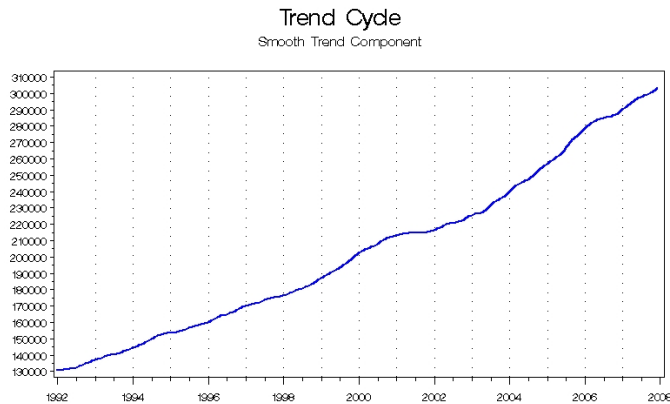


Figure 1: Smooth Trend

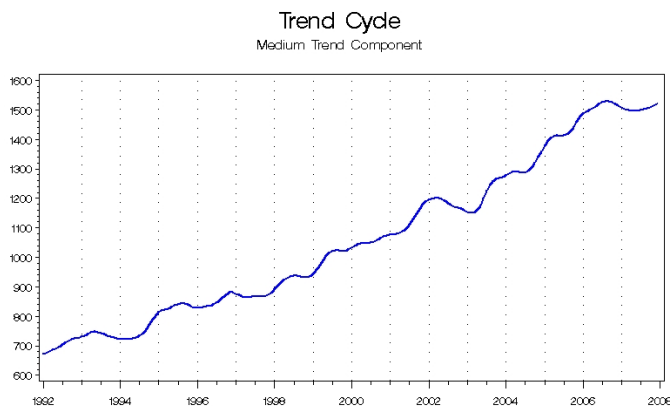


Figure 2: Medium-Complexity Trend

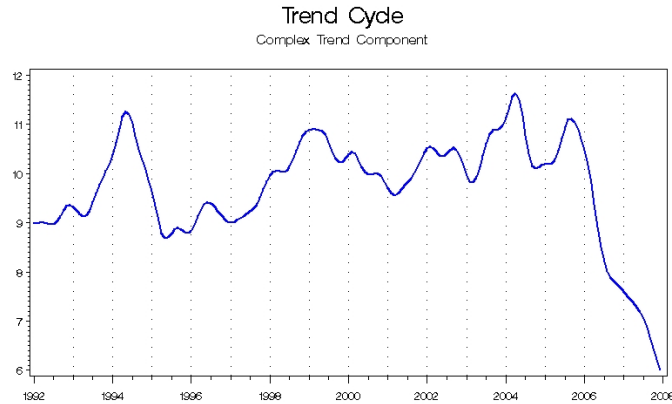


Figure 3: Complex Trend

Because of our simulation method, we tested for the presence of a form of the trading day effect that does not include an adjustment for the average length of February (or of the first quarter). More details about the trading day adjustments are available in the X-13A-S manual (U. S. Census Bureau 2009). This elimination of the length-of-month effect meant that the models for our simulated series were nested.

Because the simulated series were simpler than the real series in that they had no Easter effects and few outliers, we could complete additional comparisons by running all series with and without trading day effects. We computed the AIC_C differences for both the monthly and quarterly series and compared the differences for each series. This comparison showed whether the strength of the effects in the monthly series affected the strength of the effects in the quarterly series.

5.2 Simulated Series: Results

Initially AIC_C identified trading day effects in 50% (9 of 18) of the series that had imposed trading day effects. Also, it identified a trading day effect where there were no imposed effects for the series with medium trend and medium-range seasonal component. When we added the trading day regressor to the model for significance comparisons, there were additional outliers chosen for three series; all three were composed of the complex trend and the large seasonal component, but the new outliers did not result in changes in trading day identification. One series did have a change in AIC_C result after final model reestimation.

No trading day effects were identified in series composed of the complex trend, and there was no difference by the type of imposed trading day effects. Table 6 shows results for the 18 series that had imposed trading day effects.

The results of the type of imposed trading day effects make some sense. The series with no imposed trading day effects had only one instance of a falsely-identified effect, for the combination of medium-complex trend and medium seasonal factor range. The complex trends may have made it more difficult to identify the trading day effects. Surprisingly, the not-found trading day effects were identified just as often and in the same circumstances as the found effects.

These results may help users decide when it might be reasonable to try to fit trading day effects for quarterly series. If the trend component is especially complex, it might not be possible to identify trading day effects. It seems apparent that the AIC_C diagnostic is not

highly likely to identify trading day effects when not present, however, the significance of the effects as measured by the regression chi square p value was misleading, as it was 0.00 for 44% (4 of 9) of the series with no imposed effects.

Table 6: Simulated Series With AIC_C Identifying Trading Day Effects for Smooth and Medium Trends by Seasonal Component Size					
		Smooth Trend	Medium Trend	Complex Trend	Total
Seasonal Range	Small	2	0	0	2
	Medium	2	2	0	4
	Large	2	2	0	4
Total		6	4	0	10 of 18 (56%)

Because of our simulation procedure, the monthly estimated regression coefficients were very close to the values we imposed. The quarterly estimated coefficients were not especially close to the monthly coefficients. Although correlations were high for the estimated Saturday coefficients, the quarterly estimated coefficients were less than the monthly estimates for all nine series. Table 7 shows the correlations.

Table 7: Correlations of Monthly and Quarterly Estimated Trading Day Regression Coefficients		
	Quarterly Effects Identified	Quarterly Effect Nots Identified
Monday	-0.16	-0.01
Tuesday	0.44	0.48
Wednesday	0.53	0.28
Thursday	0.30	0.18
Friday	0.42	0.26
Saturday	0.91	0.87
Sunday	0.06	0.14

Of the eight series that had imposed trading day effects whose AIC_C results indicated no effects were present, the AIC_C differences ranged from -1.85 to -11.69. Table 8 shows the distribution of these differences.

Table 8: Range of AIC_C Differences for Quarterly Series With No Identified Trading Day Effects		
AIC _C Difference	Number of Series	
-2 ≤ Diff < 0	13%	1
-6 ≤ Diff < -4	38%	3
Diff < -8	50%	4
Total	8	

Comparisons of the monthly and quarterly AIC_C differences provided some insight into the strength of the identified trading day effects. Figure 4 is a scatterplot of the quarterly AIC_C differences (y-axis) vs. the monthly AIC_C differences (x-axis) for the nine series that had no

imposed trading day effects. Figure 5 shows the nine series with the not-found trading day effects. Figure 6 shows the nine series with the found trading day effects. In each plot, positive AIC_C differences indicate a preference for trading day effects. The six largest differences seen in the monthly series were greater than 500 and all were for series simulated with the small seasonal factor range. The two largest differences for the quarterly series also were for series simulated with the small seasonal factor range, but the five largest differences were for series simulated with the smooth trend component. For the four series simulated with the small seasonal factor range but composed of medium or complex trend component, despite the incredibly strong preference for trading day effects in the monthly series, the effects were not identified in the quarterly series.

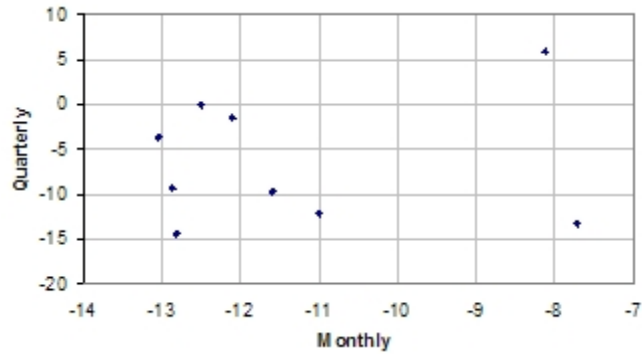


Figure 4: Scatterplot of Quarterly and Monthly AIC_C Differences for Simulated Series With No Trading Day Effect

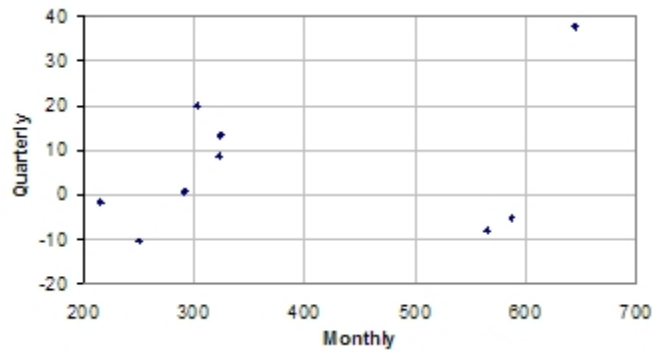


Figure 5: Scatterplot of Quarterly and Monthly AIC_C Differences for Simulated Series With the Not-Found Trading Day Effect

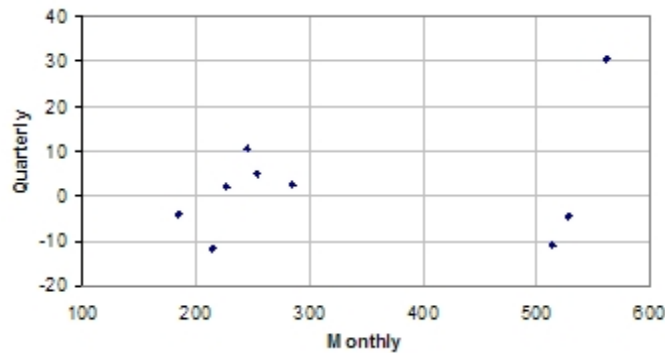


Figure 6: Scatterplot of Quarterly and Monthly AIC_C Differences for Simulated Series With the Found Trading Day Effect

6. Conclusions

It seems clear that even with some expectation that a series may have trading day effects, those effects may not be identifiable in quarterly data: even very strong trading day effects apparent at the monthly level may not be detected at the quarterly level. Some quarterly series do have strong, identifiable trading day effects, and probably the complexity of the trend component and size of the irregular component play a role, but it still is not completely clear why some trading day effects are identifiable and others are not.

7. Future Work

There are many additional issues related to quarterly trading day effects that we would like to investigate. Additional simulations of trading day effects of varied magnitudes and deliberate emphasis on the different days of the week might shed light on what types of effects are more likely to be identified. Additional simulations with varied behavior of the irregular component also are necessary.

In addition, although there are only 12 published Building Permits series, longer series are available, and the trading day effects may have remained very similar over time because of the schedules of building permit offices. We would like to check whether longer series are more likely to have identifiable effects. For series like Retail trade where store hours and shopping patterns have changed over time, this type of investigation may not be as helpful.

Also, for this study we concentrated on the six-regressor trading day effect. A one-regressor effect is available to estimate effects that vary only between weekdays (Monday through Friday) and weekends (Saturday and Sunday). Simulations using this simpler regressor may provide further insight.

Investigations using the new spectral frequencies from Ladiray (2008) may shed more light on the identification of quarterly trading day effects as well.

Acknowledgements

The authors are grateful to Rita Petroni, Amy Newman-Smith, Bill Bell, and Brian Monsell for their helpful suggestions.

Authors' Note

Much of the data analysis for this paper was generated using Base SAS® software, SAS/AF® software, and SAS/GRAPH® software, Versions 8 and 9 of the SAS System for Windows. Copyright © 1999-2003 SAS Institute Inc. SAS and all other SAS Institute Inc. product or service names are registered trademarks or trademarks of SAS Institute Inc., Cary, NC, USA. We performed additional analysis using Microsoft® Excel 2000. Copyright © 1985-1999 Microsoft Corporation.

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