Issues in Estimating Easter Regressors Using RegARIMA Models with X-12-ARIMA

David F. Findley, Kellie Wills, and Brian C. Monsell
Contact-Brian Monsell, (brian.c.monsell@census.gov)
U. S. Census Bureau

Key Words: moving holiday effects, revisions, outliers, seasonal adjustment

Disclaimer

This report is released to inform interested parties of ongoing research and to encourage discussion of work in progress. The views expressed on statistical, methodological, technical, or operational issues are those of the authors and not necessarily those of the U.S. Census Bureau.

1. Introduction

The most common moving holiday effect found in U. S. economic flow series is the Easter effect. For many retail sales series, levels of sales are elevated in the period just before the Easter holiday (which varies between March 22 and April 25). Because of this, X-12-ARIMA has long had a built-in regressor corresponding to the Easter holiday.

The Easter regressor within X-12-ARIMA follows the simplest model of Bell and Hillmer (1983); it assumes that the level of activity changes on the $w-$th day before the holiday for a specified w, and remains at the new level until the day before the holiday. For a given effect window w, the Easter regressor $E(w,t)$ is generated as

$$E(w,t) = \frac{1}{w} \times [\text{no. of the } w \text{ days before Easter falling in month } t] - \mu_{w,t}$$

where $\mu_{w,t}$ are the long-run monthly (or quarterly) means of the first part of the $E(w,t)$ equation corresponding to the first 400-year period of the Gregorian calendar, 1583-1982. The $\mu_{w,t}$ capture the seasonal component, and thus their removal yields a regressor $E(w,t)$ which does not estimate effects belonging to the seasonal component of the series. This enables the seasonal factors of the series to be estimates of all seasonal movement in the series. Table 1 gives the value of $\mu_{w,t}$ for commonly used values for w.

In the X-12-ARIMA input, $E(w,t)$ is specified as `easter[w]`; this is how Easter regressors will be referred to in this document.

An earlier study, Findley and Soukup (2000), shows how comparing AIC values and analyzing graphs of out-of-sample forecast errors can be used to determine if regARIMA models should include moving holiday terms.

This study seeks to provide guidance for practical concerns analysts have when including Easter regression effects in regARIMA models for economic time series. We seek to answer the following questions regarding the use of Easter regressors in regARIMA models of economic data as they are incorporated into X-12-ARIMA:

- How many years of data are needed to detect Easter holiday effects with high reliability?
- How many years of data are needed to obtain useful estimates of the Easter effect (estimates that improve the seasonal adjustment)?
- How often does X-12-ARIMA produce "false positives," i.e. select an Easter effect when none exists? What is the impact of March and April outliers on the number of false positives?
- How often does X-12-ARIMA misidentify the Easter effect length, e.g. select easter[15] when the true effect is easter[8]?
- How does the estimation of trading day effects affect Easter effect detection?

These questions were investigated using synthetic series with Easter effects constructed to conform to the models assumed by X-12-ARIMA.

In addition, we examine an alternate model for Easter effects in Section 8. This model will incorporate Easter Monday into the estimated Easter effect.

2. Construction of series with known Easter effects

We used 30 simulated series, representing 12 years of monthly data (144 observations) that were constructed for an earlier paper (Hood, Ashley, and Findley 2000).[1] Each series combines a known trend, seasonal factor and irregular, as described in Table 2. The trends designated `st1`, `st2`, and `st3` come from SEATS adjustments of three different series, while the trends designated `xt1`, `xt2`, and `xt3` come from the X-12-ARIMA adjustments of the same three series. Designations for seasonal factors are similar. SEATS trends were combined with X-12-ARIMA seasonal components, and vice versa. Hood, Ashley, and Findley (2000) sampled irregulars from pooled SEATS and X-12-ARIMA irregulars. The irregulars used for the original study appeared less random (showed more local autocorrelation) than one would expect from a typical X-12-ARIMA or SEATS irregular. Therefore we created new irregulars in the same size ranges by resampling from the same files Hood, Ashley, and Findley (2000) used to sample the original irregulars.

We created series with small (0.01 coefficient), moderate (0.03 coefficient) and large (0.08 coefficient) Easter effects for each of

[1] Hood, Ashley, and Findley (2000) also consider 18 synthetic series with an extremely large irregular factor range (0.4 - 1.9). With such series, Easter effects of the size we consider are not detectable. We do not present results for these series, nor for six other series from their study that appeared to have an Easter effect we were unable to completely remove.

Easter window (w)	$\mu_{w,March}$	$\mu_{w,April}$
1	0.2350	0.7650
8	0.3534	0.6466
15	0.4695	0.5305

Table 1: 400-year means for `easter[w]` of selected window lengths w

Trend descriptions			
st /xt = SEATS trend / X-12-ARIMA trend from same series			
	st1/xt1	complex cyclical trend	
	st2/xt2	trend in between smooth and complex trend	
	st3/xt3	smooth trend	
Ranges for seasonal factors and irregular			
ss /xs = SEATS seasonal / X-12-ARIMA seasonal from same series			
Seasonal factors			
	ss1/xs1	very small	0.96-1.06
	ss2/xs2	large	0.5 -1.2
	ss3/xs3	moderate	0.85-1.12
Irregular			
	rs2	very small	0.95-1.04
	rs3	moderate	0.9 -1.13

Table 2: Trends, seasonal factors and irregulars of simulated series

	Maximum Easter factor	Minimum Easter factor
0.08 easter[8]	1.0531	0.9496
0.03 easter[8]	1.0196	0.9808
0.01 easter[8]	1.0065	0.9936
0.08 easter[15]	1.0434	0.9584
0.03 easter[15]	1.0160	0.9842
0.01 easter[15]	1.0053	0.9947
0.08 easter[1]	1.0631	0.9406
0.03 easter[1]	1.0232	0.9773
0.01 easter[1]	1.0077	0.9924

Table 3: Maximum and minimum Easter factors for 12 years ending April 2002.

three Easter effect lengths: easter[1], easter[8] and easter[15]. To impose these effects on the simulated series, we added an Easter regressor with a coefficient of -0.01, -0.03 or -0.08 to the regARIMA model for the series, and used X-12-ARIMA to remove the effect of the regressor. To adjust for the negative coefficient, X-12-ARIMA adds a corresponding effect to the series, resulting in a positive Easter effect.[2] The resulting maximum and minimum Easter factors for the 12-year period ending April 2002 appear in Table 3.

3. Detection

We investigated how reliably X-12-ARIMA detected the simulated Easter effects created in the series above. The main test used by X-12-ARIMA utilizes AICC, which is a version of Akaike's Information Criterion (also called the F-corrected AIC) which contains a correction for sample size. Among competing models for a given time series, the model with the smallest AICC value is the model preferred by the criterion. For more information on AICC, see Hurvich and Tsai (1989).

When performing the AICC test, X-12-ARIMA estimates the regARIMA model separately with each of the regressors easter[1], easter[8] and easter[15], and without an Easter regressor. It then selects the regressor yielding the smallest AICC or the model without Easter if its AICC is smallest.

To obtain the ARIMA part of the model, we used the automatic model identification capability in version 0.3 of X-12-ARIMA (Build 154, compiled September 6, 2005), also with automatic outlier detection. As part of this procedure, a final check of the t-statistic generated from the Easter regressor is performed, and the program rejects the Easter regressor if the t-statistic indicates the effect is not significant. For more information on the automatic model identification procedure in version 0.3 of X-12-ARIMA, see Monsell (2002).

3.1 Years of data required for detection

Table 4 gives the number of series (out of 30) for which the automatic model identification procedure of X-12-ARIMA selects a model without an Easter regressor, when the series in fact contains the Easter effect indicated in the column heading. The time period is the number of years in the row heading, ending April 2002. Table 4 also shows that large/moderate effects (coefficient 0.08/0.03) are detectable with high reliability with four years of data. In the period ending April 2002, similar success for small (1%) effects

[2] Note that the actual imposed Easter effect is smaller than the coefficient would imply, because X-12-ARIMA deseasonalizes the Easter effect.

can be achieved with twelve years of data. However, Section 3.2 illustrates that detection of the 1% effect for eight years of data or less depends on the specific sequence of Easter dates considered.

Table 5 gives the number of series (out of 30) for which the X-12-ARIMA AICC test alone (without the additional check of the t-statistic of the Easter regressor) selects a model without an Easter regressor; note that the number of rejections is noticeably lower in most cases for the small Easter effect.

3.2 How dates of Easter affect detection

We investigated how detection was affected by the sequence of Easter dates in the years we chose to associate with the simulated data. We considered several eight- and six-year spans of data over the 12-year period of the study. Table 6 provides the dates of Easter for reference.

Tables 7, 8 and 9 present the number of series (out of 30) for which the automatic model identification procedure rejects all Easter regressors (selecting a model without Easter), when the series in fact contains easter[8] (Table 7), easter[15] (Table 8) or easter[1] (Table 9). The tables also present the number of rejections of a 0.01 coefficient Easter effect when March 1993 is included in the regression as an additive outlier. Visual inspection of the rs3 irregular indicates that March 1993 is an outlier in the 15 series having this irregular, but has a t-statistic below the threshold set by X-12-ARIMA for automatic outlier detection, and is thus not adjusted for; see Section 5. In the series with a 0.01 coefficient Easter effect and the rs3 irregular, the t-statistics for this outlier ranged from 2.33 to 3.17 with 12 years of data, compared with the cutoff of 3.89 used by X-12-ARIMA.

To assess the effect of outliers in April, we constructed additive outliers for April 1998. (This date was chosen for the included outlier because each of the 6-year spans considered included it.) The outlier regression coefficient of 0.04 (and -0.04) was selected so that the t-statistics for the April 1998 outlier would be comparable to those for the March 1993 outlier. Tables 7, 8 and 9 illustrate the profound effect of this outlier on detection.

The tables indicate that while detection of the 0.03 coefficient Easter effect is insensitive to Easter date, detection of the 0.01 coefficient effect is sensitive to both dates and the presence of outliers.

4. Estimation

We investigated the accuracy of Easter effect estimation by comparing seasonal adjustments with and without an Easter regressor to the series adjusted for the correct (synthetic) seasonal factors (the "true adjusted series"). For series containing an Easter effect, adjustment with a good estimate of the Easter coefficient should consistently produce results closer to the true adjusted series than adjustment without an Easter regressor. However, if there are not enough data to accurately estimate the Easter coefficient, the adjustment with Easter may differ more from the true adjusted series than the adjustment without Easter. Since the original series were constructed from known trend, seasonal and irregular components, we were able to obtain the true adjusted series by dividing the original series (without an Easter effect) by the seasonal component. We used X-12-ARIMA to seasonally adjust each series both with an Easter effect (either of the true length or of an incorrect length), and with no Easter regressor. For each adjustment, we calculated a relative root mean squared deviation (RRMSQD) and a relative mean absolute deviation (RMAD) from the true seasonally adjusted series, as shown below:

$$RRMSQD = \sqrt{N^{-1} \sum_{t=1}^{N} \frac{(x_t - \widehat{x}_t)^2}{x_t^2}},$$

$$RMAD = N^{-1} \sum_{t=1}^{N} \frac{|x_t - \widehat{x}_t|}{x_t},$$

where x_t is the true adjusted series and $\widehat{x}_t$ is the estimated adjusted series. Only the March and April values of the seasonally adjusted series were used in the calculation of the RRMSQD and RMAD. The adjustment with the smaller RRMSQD or RMAD was considered to be the better adjustment.

Table 10 gives the average March/April RRMSQD and RMAD for all 30 adjusted series. The columns labeled "Easter better" give the number of series for which the adjustment with Easter was the better adjustment. The table shows that moderate effects (0.03 coefficient) are estimable with four years of data when adjusting with the correct Easter length (8). Small effects (0.01 coefficient) are estimable with eight years of data for most series when adjusting with the correct Easter length. However, the reduction in RRMSQD and RMAD produced by estimating the Easter effect is very small for these small effects.

We checked whether misidentifying the Easter length, e.g. adjusting with easter[15] when the true Easter is easter[8], would lead to worse adjustments. With 12 years of data, for all series, adjusting an 0.03 easter[8] effect as easter[15] results in a better adjustment than adjusting without an Easter regressor. (For results on misidentification of an 0.03 easter[8] effect with 12 years of data, please see Section 6.) The results for adjusting 8 years of data with a true 0.01 easter[8] effect are surprising. Adjusting with an incorrect easter[15] length results in more series with an improved adjustment than adjusting with the correct length, easter[8]. We examined the Easter coefficient for the four series for which easter[8] gives a worse adjustment than no Easter, while easter[15] gives a better adjustment than no Easter. In each case, the easter[15] coefficient was closer to the true 0.01 value than the easter[8] coefficient. It would appear that a better estimate of the Easter coefficient leads to a better adjustment in these cases.

5. Rates of false detection

We used the original simulated series without Easter effects added to investigate false positive results, i.e. how often X-12-ARIMA detected an Easter effect not present in the data. With as few as four years of data for the period ending April 2002, X-12-ARIMA produced no false positives.

However, additive outliers in March and April readily induce false detections. For the twelve years of data ending April 2002, adding two such outliers (in April 2000 and March 2002) was sufficient to cause false Easter effect identifications for 23 of 30 series.

Rejections (among 30 series) for period ending April 2002

	easter[8]			easter[15]			easter[1]		
	0.08	**0.03**	**0.01**	**0.08**	**0.03**	**0.01**	**0.08**	**0.03**	**0.01**
12 yr	0	0	0	0	0	1	0	0	0
10 yr	0	0	10	0	0	11	0	0	7
8 yr	0	0	5	0	0	7	0	0	3
6 yr	0	0	9	0	0	11	0	0	8
4 yr	0	0	30	0	0	29	0	0	30

Table 4: Rejections of Easter effect when effect was present (periods end April 2002).

AICC rejections (among 30 series) for period ending April 2002

	easter[8]			easter[15]			easter[1]		
	0.08	**0.03**	**0.01**	**0.08**	**0.03**	**0.01**	**0.08**	**0.03**	**0.01**
12 yr	0	0	0	0	0	0	0	0	0
10 yr	0	0	2	0	0	3	0	0	1
8 yr	0	0	0	0	0	2	0	0	0
6 yr	0	0	7	0	0	8	0	0	7
4 yr	0	0	24	0	0	24	0	0	28

Table 5: Rejections of Easter effect when effect was present, using the AICC test alone with no check of the Easter regressor's t-statistic (periods end April 2002).

The outliers were generated with a coefficient of 0.04, a choice for which their t-statistics are below the threshold set by X-12-ARIMA for automatic outlier detection.

6. Misidentifications

We investigated how often the moderate (0.03 coefficient) Easter effect was detected but misidentified, e.g. AICC prefers easter[15] for a series with known easter[8], in periods ending April 2002.

With 12 years of data, 14 of 30 series with easter[8] with a known 0.03 coefficient were misidentified as easter[15] (Table 11). With 10 years of data, 12 of the 30 series were misidentified. However, with 8 years of data, the number of misidentifications dropped to zero. All the misidentified series had the rs3 irregular, and visual inspection of this irregular suggested that March 1993 might be an outlier. When March 1993 was included as an additive outlier in the regression for series that were misidentified, no series were misidentified with 10 or 12 years of data. The t-statistics for the March 1993 additive outlier ranged from 2.36 to 3.06 with 12 years of data. (These t-statistics are below the threshold set by X-12-ARIMA for automatic outlier detection.) Adjusting the misidentified series with the inappropriate easter[15] regressor would still result in a better adjustment than adjusting without an Easter regressor, as noted in Section 3.

With 6 years of data, 17 of 30 series with known easter[15] were misidentified as easter[8], and one of the series was misidentified as easter[1]. Most of the misidentified series had the rs2 irregular. For three of these series, March 1997 had a t-statistic greater than 2 when included as an additive outlier regressor. In all three cases, easter[15] was correctly identified when the additive outlier was included.

We found no misidentifications of easter[1] in any length time period ending April 2002.

7. Easter detection when a trading day regressor is estimated

Since many Census economic series are affected by both Easter holiday and trading day effects, we investigated what impact estimating the trading day effect has on Easter detection. To 30 series having a moderate (0.03 coefficient) easter[8] effect, we added a one-coefficient (weekday/weekend) trading day effect ("`td1coef`" in X-12-ARIMA) with an 0.03 coefficient. We specified the td1coef regressor and allowed X-12-ARIMA to perform the AICC test for an Easter effect.

With eight years or more of data (ending April 2002), AICC correctly accepts the Easter effect for all 30 series. At least eight years of data are recommended for accurate estimation of trading day effects (Findley, Monsell, Bell, Otto, and Chen 1998). With six years of data, AICC correctly accepts the Easter effect for all series. However, with four years of data, AICC rejects the Easter effect for 19 series. (By comparison, when no trading day effect is present, AICC accepts the Easter effect for all 30 series.)

8. An alternate Easter regressor

Many countries around the world recognize Easter Monday as a national holiday. For these countries, the Easter regressor within X-12-ARIMA can give inadequate results. This is because there is often increased economic activity before Good Friday but decreased activity from Good Friday through Easter Monday. To alleviate this, we consider an alternate form of the Easter regressor based on research for Australian time series given in Zhang, McLaren, and Leung (2003).

Dates of Easter			
31st March 1991	19th April 1992	11th April 1993	3rd April 1994
16th April 1995	7th April 1996	30th March 1997	12th April 1998
4th April 1999	23rd April 2000	15th April 2001	31st March 2002

Table 6: Dates of Easter in the periods considered in Tables 7, 8 and 9

easter[8] - Rejections (of 30)

	0.03	**0.01**	**0.01 (AO1993.Mar in regression)**	**0.01 (AO1998.Apr, b = 0.04)**	**0.01 (AO1998.Apr, b = - 0.04)**
8 years ending Apr 2002	0	5	NA	0	26
8 years ending Apr 2000	0	10	1	2	30
8 years ending Apr 1998	0	6	2	0	26
6 years ending Apr 2002	0	9	NA	0	30
6 years ending Apr 2000	0	3	NA	0	27
6 years ending Apr 1998	0	14	2	2	30

Table 7: Effect of Easter date sequence and outliers on detection of easter[8]

easter[15] - Rejections (of 30)

	0.03	**0.01**	**0.01 (AO1993.Mar in regression)**	**0.01 (AO1998.Apr, b = 0.04)**	**0.01 (AO1998.Apr, b = - 0.04)**
8 years ending Apr 2002	0	7	NA	1	26
8 years ending Apr 2000	0	12	2	2	30
8 years ending Apr 1998	0	8	3	0	30
6 years ending Apr 2002	0	11	NA	0	30
6 years ending Apr 2000	0	5	NA	0	28
6 years ending Apr 1998	0	21	4	2	30

Table 8: Effect of Easter date sequence and outliers on detection of easter[15]

easter[1] - Rejections (of 30)

	0.03	**0.01**	**0.01 (AO1993.Mar in regression)**	**0.01 (AO1998.Apr, b = 0.04)**	**0.01 (AO1998.Apr, b = - 0.04)**
8 years ending Apr 2002	0	3	NA	0	22
8 years ending Apr 2000	0	8	2	1	29
8 years ending Apr 1998	0	0	0	0	23
6 years ending Apr 2002	0	8	NA	0	28
6 years ending Apr 2000	0	4	NA	0	26
6 years ending Apr 1998	0	13	3	1	29

Table 9: Effect of Easter date sequence and outliers on detection of easter[1]

			RRMSQD			RMAD		
true Easter	**years**	**adjustment**	**with Easter**	**no Easter**	**Easter better**	**with Easter**	**no Easter**	**Easter better**
0.03 easter[8]	4	easter[8]	0.0044	0.0148	30/30	0.0037	0.0130	30/30
0.03 easter[8]	12	easter[15]	0.0055	0.0149	30/30	0.0046	0.0125	30/30
0.01 easter[8]	8	easter[8]	0.0053	0.0065	25/30	0.0043	0.0052	25/30
0.01 easter[8]	8	easter[15]	0.0052	0.0065	26/30	0.0041	0.0052	28/30

Table 10: Average RRMSQD in March/April

	easter[8] 0.03	**easter[8] 0.03 AO1993.Mar**	**easter[15] 0.03**	**easter[1] 0.03**
12 yr	14[1]	0	0	0
10 yr	12[1]	0	0	0
8 yr	0	NA	1[2]	0
6 yr	0	NA	17[2],1[2]	0
4 yr	4[1],3[3]	NA	12[3]	0

Table 11: Misidentifications in 30 series for periods ending April 2002

[1] = misidentified as easter[15]; [2] = misidentified as easter[8];
[3] = misidentified as easter[1].

The alternate form for the Easter regressor we consider assumes that one can break the Easter effect into two parts: a pre-Easter effect of w days before Good Friday, and an Easter Holiday effect starting on Good Friday and lasting through Easter Monday. This takes the form of two regressors – a pre-holiday effect (where sales are expected to be elevated) and an effect for the duration of the holiday (where sales are expected to decline).

The pre-holiday effect is generated as

$$BE(w,t) = \frac{1}{w} \times [\text{no. of the } w \text{ days before Good Friday falling in month } t] - \mu^{BE}_{w,t},$$

and the effect during the holiday is generated as

$$DE(t) = \frac{1}{4} \times [\text{no. of days between Good Friday and Easter Monday (inclusive) falling in month } t] - \mu^{DE}_{t}.$$

where again, $\mu^{BE}_{w,t}$ and $\mu^{DE}_{w,t}$ are the "long-run" monthly (or quarterly) means used to center the respective Easter regressors.

8.1 Application to the Australian Total Retail Turnover Series

An example comparing the default X-12-ARIMA Easter effect with the alternate Easter regressors is now given using the Australian Total Retail Turnover series from Zhang, McLaren, and Leung (2003). The base model for this series was found to be an ARIMA (0 1 1)(0 1 1) model, and trading day regressors were found to be significant.

To determine which Easter regressor should be used, we use two approaches. The first utilizes likelihood-based model selection criteria by comparing the values of AICC.

In addition to `easter[`w`]`, the default X-12-ARIMA Easter regressor with window length w, we now define `easter2[`w_2`]`, the alternate Easter regressor, with a window length of w_2 for the pre-Easter effect. Table 12 shows that the alternate Easter regressor `easter2[6]` has a much lower AICC than the other regressors – so AICC prefers this regressor. Note that the window for the pre-Easter effect of the `easter2[6]` regressor assumes that the elevation in the Easter effect starts on the Saturday before Good Friday – which is the same as the `easter[8]` regressor.

Easter Regressor	AICC
No Easter	2651.4425
easter[1]	2653.5009
easter[8]	2639.0764
easter[15]	2639.4447
easter2[6]	2624.9873
easter2[13]	2634.1864

Table 12: AICC values for different Easter regressors fit to Australian Total Retail Turnover series (source: Australian Bureau of Statistics)

Another method for determining which Easter regressor to choose is to examine out-of-sample forecast error plots available from X-12-Graph (Hood 2002). X-12-ARIMA's `history` spec is used to obtain differences of the accumulating sums of squared forecast errors between the competing models for forecast leads of interest (in this case, 1 and 12). If the direction of the accumulating differences is predominantly upward, then the forecast errors are predominantly larger for the first model (in this case, the model with the X-12-ARIMA Easter regressor), and we prefer the second model.

Figure 1 shows a slight preference for the `easter2` regressor, as the increasing nature of the plot for 12-step ahead forecasts implies that forecasts for the model with `easter2` have lower out-of-sample forecast error than those of X-12-ARIMA's Easter regressor, though the difference in the forecast errors appears to level off towards the end of the series. It's troubling to see the blips in the 12-step ahead forecast error graph in March and April of 1997 and 1999 - they seem to imply that the model with the `easter2` regressor does a better job of forecasting for March, while the model with the traditional X-12-ARIMA regressor is slightly better for April in those years.

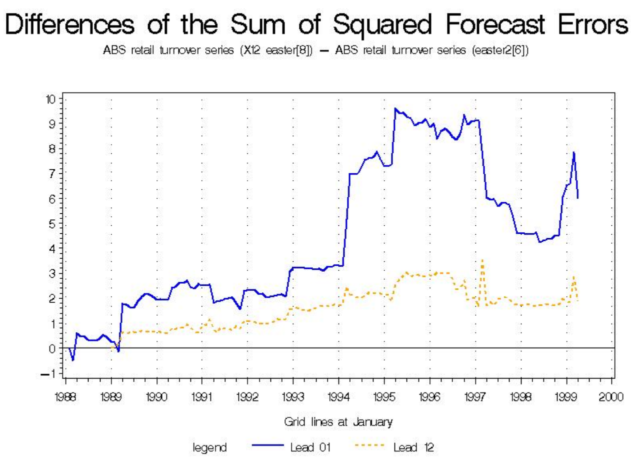

Figure 1: Forecast error plot comparing forecasts from models with different Easter regressors for Australian Total Retail Turnover (source: Australian Bureau of Statistics)

For more information on forecast error plots, see Findley and Soukup (2000) and Findley (2005); for an application to Italian economic series, see Findley and Hood (1999).

The alternate Easter regressors and regressors for other Lunar holiday effects (Lin and Liu 2002) described in this section can be generated easily using the Census Bureau's GENHOL program. This program is available on the Census Bureau website: http://www.census.gov/srd/www/x12a/ .

9. Conclusions

We can take several points away from our examination of simulated series:

- Large/moderate effects (coefficient 0.08/0.03) are detectable with four years of data.
- Detection of the 0.01 coefficient effect depends on the sequence of Easter dates. Detection is sensitive to March and April outliers.
- Moderate effects (0.03 coefficient) are estimable with four years of data. Small effects (0.01 coefficient) are estimable with eight years of data for most series.
- With as few as four years of data for the period ending April 2002, X-12-ARIMA produced no false positives for the Easter effect. However, two outliers in March/April are sufficient to induce false positives in most series.
- Outliers with relatively small t-statistics can result in misidentification of Easter effect length.
- Estimating a trading day effect has some impact on Easter detection for short series, possibly because there are not enough data for accurate estimation of the trading day effect.

We have also shown an example of an alternate Easter regressor that could prove useful for modeling series from areas where Easter Monday is recognized as a holiday.

10. Acknowledgements

We thank Kathleen McDonald-Johnson and Donald Martin for their careful reading of drafts of this document.

The forecast error graph shown in this paper was generated using SAS software (Copyright, SAS Institute Inc). SAS and all other SAS Institute Inc. product or service names are registered trademarks or trademarks of SAS Institute Inc., Cary, NC, USA.

References

Bell, W. R. and Hillmer, S. C. (1983), "Modeling Time Series With Calendar Variation," *Journal of the American Statistical Association*, 78, 526–534.

Findley, D. F. (2005), "Asymptotic stationarity properties of out-of-sample forecast errors of misspecified regARIMA models and the optimality of GLS for one-step-ahead forecasting," *Statistica Sinica*, 15, 447–476.

Findley, D. F. and Hood, C. C. (1999), "X-12-ARIMA and Its Application to Some Italian Indicator Series," in *Seasonal Adjustment Procedures – Experiences and Perspectives*, Rome: Istituto Nazionale di Statistica (ISTAT), pp. 231–251, http://www.census.gov/ts/papers/x12istat.pdf.

Findley, D. F., Monsell, B. C., Bell, W. R., Otto, M. C., and Chen, B. C. (1998), "New capabilities of the X-12-ARIMA seasonal adjustment program (with discussion)," *Journal of Business and Economic Statistics*, 16, 127–77, http://www.census.gov/ts/papers/jbes98.pdf.

Findley, D. F. and Soukup, R. J. (2000), "Modeling and Model Selection for Moving Holidays," *Proceeding of the American Statistical Association, Business and Economic Statistics Section*, 102–107, http://www.census.gov/ts/papers/asa00_eas.pdf.

Hood, C. C. (2002), "X-12-Graph: A SAS/GRAPH Program for X-12-ARIMA Output, User's Guide for the X-12-Graph Interactive for PC/Windows, Version 1.2," U. S. Census Bureau, U. S. Department of Commerce.

Hood, C. C., Ashley, J. D., and Findley, D. F. (2000), "An Empirical Evaluation of the Performance of TRAMO/SEATS on Simulated Series," American Statistical Association, Proceedings of the Business and Economic Statistics Section http://www.census.gov/ts/papers/asa00_ts.pdf.

Hurvich, C. M. and Tsai, C. (1989), "Regression and Time Series Model Selection in Small Samples," *Biometrika*, 76, 297–307.

Lin, J.-L. and Liu, T.-S. (2002), "Modeling lunar calendar holiday effects in Taiwan," *Taiwan Economic Forecast and Policy*, 1–37, http://www.census.gov/ts/papers/lunar.pdf.

Monsell, B. C. (2002), "An Update on the Development of the X-12-ARIMA Seasonal Adjustment Program," *Proceedings of the 3rd International Symposium on Frontiers in Time Series Modeling*, Institute of Statistical Mathematics, Tokyo, pp. 1–11.

Zhang, X., McLaren, C. H., and Leung, C. C. S. (2003), "An Easter proximity effect: Modelling and adjustment," *Australian and New Zealand Journal of Statistics*, 43, 269–280.