MODELING TIME-VARYING TRADING-DAY EFFECTS IN MONTHLY TIME SERIES

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1. INTRODUCTION

Trading-day effects reflect variations in monthly time series due to the changing composition of months with respect to the numbers of times each day of the week occurs in the month. A relevant question regarding trading-day effects is whether they remain constant over time? This is especially pertinent for retail sales data in which trading-day effects presumably depend on consumers' shopping patterns and on hours that retail stores are open, two things that have changed over time in the U.S. Seasonal adjustment practitioners sometimes deal with this issue by restricting the length of the series to which the trading-day model is fit. However, this can provide only a crude approximation to trading-day effects that vary through time. In this paper we explore some alternative models for time-varying trading-day effects and investigate possible time variation in trading-day effects in some Census Bureau monthly time series.

Monsell (1983), Dagum, Quenneville and Sutradhar (1992, hereafter DQS), and Dagum and Quenneville (1993, hereafter DQ) considered stochastic models for time-varying trading-day coefficients. Monsell used random walk models for the coefficients; DQS and DQ considered a more general model in which applying some order of differencing (not just first order) to the trading-day coefficients yields white noise. A limitation to the analysis of Monsell and of DQS is that they considered just trading-day plus white noise irregular models. Monsell applied this model to simulated data, DQS to data filtered by X-11 irregular filters to remove trend and seasonality. DQ considered a more general model including seasonal, trend and irregular components in addition to time-varying trading-day, though for the example presented they chose a model with fixed trading-day coefficients. Harvey (1989) and Bell (2004) considered models for time-varying trading-day coefficients in general contexts, and we discuss their models in more detail in later sections.

In the next section we review a model for fixed trading-day effects and then discuss some alternative models that allow for stochastically time-varying trading-day coefficients. Section 3 discusses aspects of fitting the models. Section 4 gives results of fitting the models to some Census Bureau time series. The final section gives conclusions and also discusses directions for future research.

2. MODELS FOR TRADING-DAY EFFECTS

Monthly time series that are accumulations of daily values (flow series) are often affected by the day-of-week composition of the month, i.e., by which days occur five times and which days occur four times in the month. Let y_t be the observed value for month t (or its logarithm) in a monthly flow time series. A basic model incorporating trading-day effects is

$$y_t = TD_t + Z_t, \qquad (1)$$

(Bell and Hillmer 1983, hereafter BH) where TD_t is the trading-day effect, and the remainder series Z_t follows some time series model such as an ARIMA (autoregressive-integrated-moving average) model (Box and Jenkins 1976) or an ARIMA components model (Harvey 1989).

We will use linear regression models for TD_t. We first consider a basic model with fixed coefficients, and then discuss alternative ways that this model can be generalized to allow for stochastically time-varying trading-day coefficients. We could include additional regression effects (other than for trading day) in the model (1), such as Easter holiday effects as in BH, but we ignore that possibility in this section to focus attention on alternative models for TD_t. In the examples of Section 4 we bring in some additional regression effects and the time series model for Z_t.

2.1 A model for fixed trading-day effects

Let the average effect of day i on the monthly value of the series be α_i, $i = 1, 2, ..., 7$, so α_1 is the average effect on the series of Monday, α_2 the average effect of Tuesday, …, and α_7 the average effect of Sunday. For month t, let D_{1t} be the number of Mondays in the month, D_{2t} the number of Tuesdays, …, and D_{7t} the number of Sundays. Also, let

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$N_t \equiv \sum_{i=1}^{7} D_{it}$ denote the length of month t (28, 29, 30, or 31), and let $\bar{\alpha} \equiv (1/7)\sum_{i=1}^{7}\alpha_i$ denote the average daily effect. To model fixed trading-day effects, BH start with

$$\sum_{i=1}^{7}\alpha_i D_{it} = \bar{\alpha} N_t + \sum_{i=1}^{7}(\alpha_i - \bar{\alpha})D_{it} . \qquad (2)$$

In equation (2) the term $\bar{\alpha}N_t$ is a length-of-month effect that we do not consider as part of the trading-day effect, and in fact drop it from the model for reasons discussed below. Thus, TD_t is given by the second term on the right hand side of (2). Since $\sum_{i=1}^{7}(\alpha_i - \bar{\alpha}) = 0$, TD_t may be written as

$$TD_t = \sum_{i=1}^{6}(\alpha_i - \bar{\alpha})(D_{it} - D_{7t}) = \sum_{i=1}^{6}\beta_i T_{it} , \qquad (3)$$

where $\beta_i = \alpha_i - \bar{\alpha}$ and $T_{it} = D_{it} - D_{7t}$, $i = 1,2,...,6$. The parameters β_i measure the differences between the Monday, … , Saturday effects and the average daily effect, $\bar{\alpha}$. The difference between the Sunday effect and the average daily effect is given by $\beta_7 \equiv \alpha_7 - \bar{\alpha} = -\sum_{i=1}^{6}\beta_i$. In equation (3) we use Sunday as the day of reference, however, clearly any of the other six days could be so used. Notice that if α_i, $i = 1,2,...,7$ are equal, the trading-day effect is zero. Also, for a non-leap year February, $D_{it} = 4$, $i = 1,2,...,7$, and the trading-day effect is zero.

Rather than include the term $\bar{\alpha}N_t$ of (2) in our models, we shall instead divide the original series (before taking logarithms) by N_t. This is consistent with the default option in the X-12-ARIMA program (Findley, Bell, Monsell, Otto and Chen 1998). We do this for the following reasons: (a) Bell (1984) noted that the length-of-month effect, $\bar{\alpha}N_t$, can be decomposed into a level effect, a fixed seasonal effect, and a leap-February effect. The first two will be accounted for by differencing in the models. Thus, the term $\bar{\alpha}N_t$ serves essentially to model leap-February effects. (b) When y_t represents logarithms of the original series, we would expect $\bar{\alpha}$ to be approximately log(29/28) = 0.035, in which case it can be shown that including $\bar{\alpha}N_t$ in the model is approximately equivalent to dividing the original time series (before taking logs) by N_t. (c) Including $\bar{\alpha}N_t$ in the model and estimating $\bar{\alpha}$ sometimes yields implausible values, i.e., values that differ substantially from 0.035.

2.2 Models for stochastically time-varying trading-day coefficients

To adapt our trading-day model to allow for stochastically time-varying coefficients, we let α_{it}, $i = 1,2,...,7$ be the effect of day i in month t, and $\beta_{it} \equiv \alpha_{it} - \bar{\alpha}_t$, $i = 1,2,...,7$ be the difference between the effect for day i in month t and the average weekly effect for the month, $\bar{\alpha}_t \equiv (1/7)\sum_{i=1}^{7}\alpha_{it}$. By definition

$$\sum_{j=1}^{7}\beta_{jt} = \sum_{j=1}^{7}(\alpha_{jt} - \bar{\alpha}_t) = 0 , \qquad (4)$$

and thus one of the coefficients can be computed as a function of the other six. With these time-varying coefficients, (3) becomes

$$TD_t = \sum_{i=1}^{6}\beta_{it}T_{it} = T_t'\beta_t , \qquad (5)$$

where β_t and T_t are column vectors with components $\beta_t = (\beta_{1t}, \beta_{2t}, \ldots, \beta_{6t})'$ and $T_t = (T_{1t}, T_{2t}, \ldots, T_{6t})'$. To complete the specification we need to give a model for β_t, or specify a joint model for the α_{it} and derive the implied model for β_t.

We shall use a random walk model to allow for time variation in the trading-day effects. We choose the random walk model because it is both simple, and nonstationary. It also includes a fixed coefficient as a special case when the random walk innovation variance is zero. A simple model seems to be required because we are not likely to be able to estimate well an involved model for the stochastic trading-day coefficients. We desire a nonstationary model because use of a stationary model, such as a stationary autoregressive model of order one (AR(1)), would imply erratic time variation in the trading-day coefficients. In fact, unless the AR parameter in the stationary AR(1) model is extremely close to one, the trading-day coefficients would show erratic variation around the fixed means, and the model also would not allow for consistent movement up or down over long periods. Hannan (1964) made an analogous observation in the context of using stationary AR models for a time-varying seasonal component. The random walk model is more appealing, as it will allow the coefficients to change more smoothly over time and will not tie them to fixed means.

Bell (2004) made a straightforward generalization of (3) by assuming that the β_{it} follow independent random walk models:

$$(1-B)\beta_{it} = \eta_{it} , \; i = 1,2,\ldots,6 , \qquad (6)$$

where B denotes the backshift operator $(Bx_t = x_{t-1})$, and the η_{it} are mutually independent white noise series with variances σ_i^2. As before, the reference day in (6) is Sunday with β_{7t} determined from (4) as

$$\beta_{7t} = -(\beta_{1t} + \cdots + \beta_{6t}) . \quad (7)$$

Though the reference day is usually chosen to be Sunday, in principle we could change the model (6) to allow any given day to serve as the reference day. The model (6) has the disadvantage that the β_{it} corresponding to the reference day has different statistical properties than the coefficients corresponding to the other six days. To see this with Sunday as the reference day, note that applying the difference operator $1-B$ to both sides of (7) gives

$$\eta_{7t} \equiv (1-B)\beta_{7t} = -(\eta_{1t} + \cdots + \eta_{6t}) . \quad (8)$$

Thus, although $\eta_{1t},\ldots,\eta_{6t}$ are assumed independent of one another, η_{7t} will be correlated with all of them (except if $Var(\eta_{it}) = 0$ for some i, in which case $\eta_{it} = 0$ almost surely). Analogous results obviously hold if we change the reference day. How much effect this issue has on actual estimates will be examined for the examples in Section 4.

Harvey (1989, pp. 43-44) instead generalized (3) by specifying a model for $\beta_{1t}, \beta_{2t},\ldots,\beta_{6t}$ that implicitly assumed that the α_{it} follow independent random walk models

$$(1-B)\alpha_{it} = \varepsilon_{it} ,\ i = 1,2,...,7 , \quad (9)$$

where the ε_{it} are mutually independent white noise series with *common* variance σ_ε^2. If we still define $\eta_{it} = (1-B)\beta_{it}$ as in (6) and (8), but now without assuming that the η_{it} are independent of one another, then we see from the definition of the β_{it} and from (9) that

$$\eta_{it} = (1-B)\beta_{it} = (1-B)(\alpha_{it} - \bar{\alpha}_t) = \varepsilon_{it} - \bar{\varepsilon}_t ,\ i = 1,2,...,7 , \quad (10)$$

where $\bar{\varepsilon}_t \equiv (1/7)\sum_{i=1}^{7}\varepsilon_{it}$. From (10) it follows that the η_{it}, $i = 1,2,...,7$ have mean zero, variance $(6/7)\sigma_\varepsilon^2$ and $Cov\left(\eta_{it},\eta_{jt}\right) = -\sigma_\varepsilon^2/7$, $i, j = 1,2,\ldots,7;\ i \neq j$. Thus, unlike with Bell's model, the seven innovations $\eta_{1t},\eta_{2t},\ldots,\eta_{7t}$ are correlated with one another, but all have the same statistical properties. However, the assumption of a common variance for the ε_{it}, implying the common variance $(6/7)\sigma_\varepsilon^2$ for the η_{it}, is more restrictive than the assumption of six different innovation variances in (6). This raises a question as to whether this restriction is appropriate in situations where the effects of one day may be changing faster over time than those of other days. We also note that (10) implies a multivariate random walk model for the β_{it} since the innovations η_{it} are cross-correlated. In this form, Harvey's model cannot be handled by the software we use for model fitting because the software requires independent unobserved components. We deal with this issue in the next section.

Notice that the variance-covariance matrix of $(\eta_{1t},\ldots,\eta_{7t})$ for both Bell's and Harvey's models is singular since the constraint (7) implies that $\mathbf{1}_7' Var\left(\eta_{1t},\ldots,\eta_{7t}\right)\mathbf{1}_7 = 0$, where $\mathbf{1}_7$ is a column vector of seven ones.

3. FITTING THE MODELS TO DATA

In this section we give details on fitting time series models that include either fixed or stochastically time-varying trading-day effects using the REGCMPNT program (Bell 2004). We first give background information on the RegComponent model, and then discuss the steps necessary to convert Harvey's model to this form so that it can be fit by the REGCMPNT program.

3.1 The RegComponent model

The general form of the RegComponent time series model is

$$y_t = x_t'\delta + \sum_{i=1}^{k} h_{it} z_{it} \quad (11)$$

where $x_t' = (x_{1t},\ldots,x_{rt})$ is a row vector of known regression variables at time t and δ is the corresponding column vector of fixed regression parameters. The h_{it}, $i = 1,\ldots,k$ are series of known constants that we call "scale factors" and z_{it}, $i = 1,\ldots,k$ are series of independent unobserved component series following ARIMA models.

The REGCMPNT program implements likelihood evaluation and maximization, forecasting, and signal extraction for RegComponent models. The program puts the model in state space form and uses the Kalman filter with a suitable smoother to do the calculations. See Bell (2004) for more details.

Fixed trading-day effects are handled in REGCMPNT by incorporating the trading-day variables and regression coefficients in (3) as part of the regression effects $x_t'\delta$. Bell's model (6) for time-varying trading-day coefficients is also easily handled in REGCMPNT by identifying six of the ARIMA components z_{it} in (11) with the time-varying coefficients β_{it}, $i = 1,2,...,6$, and setting the corresponding scale factors h_{it} to the trading-day variables $T_{1t},\ldots,T_{6t}$. An additional ARIMA component is necessary for the residual series Z_t in (1). An important point is that the RegComponent model, and hence the REGCMPNT program, requires that the innovations in the models for the z_{it} in (11) be independent. Since this is not true for the innovations associated with the random walk model for the β_{it} in Harvey's model (9), to fit his model special treatment is needed. We discuss this next.

3.2 Expressing Harvey's model as a RegComponent model

For notational purposes, let I_m denote an identity matrix of order m, $\mathbf{1}_m$ a column vector of m ones, and $\mathbf{0}_m$ a column vector of m zeroes. To convert Harvey's model (9) to an equivalent model in RegComponent form, we make a linear transformation of α_t to a multiple of $\bar{\alpha}_t$ and six series $\gamma_{1t}, \gamma_{2t}, \ldots, \gamma_{6t}$ following independent random walks. We do this using a 7×7 matrix G_7 of the form

$$G_7 = \begin{bmatrix} C \\ (1/\sqrt{7})\mathbf{1}_7' \end{bmatrix}.$$

Any 6×7 matrix C with rows that are orthogonal to each other and to the constant vector $\mathbf{1}_7$ will work. A convenient choice is generated by the trigonometric functions with period seven. Using these trigonometric functions, the six rows of the matrix C that we used have entries (for columns $j = 1, 2, \ldots, 7$)

$$\begin{aligned} c_{1j} &= \sqrt{2/7}\cos\left(2\pi j/7\right) & c_{2j} &= \sqrt{2/7}\sin\left(2\pi j/7\right) \\ c_{3j} &= \sqrt{2/7}\cos\left(4\pi j/7\right) & c_{4j} &= \sqrt{2/7}\sin\left(4\pi j/7\right) \\ c_{5j} &= \sqrt{2/7}\cos\left(6\pi j/7\right) & c_{6j} &= \sqrt{2/7}\sin\left(6\pi j/7\right). \end{aligned}$$

The factor $\sqrt{2/7}$ is used to normalize the rows of C to have length one. The matrix G_7 is thus orthonormal, i.e. $G_7 G_7' = G_7' G_7 = I_7$.

Define the new variables $\gamma_t = (\gamma_{1t}, \gamma_{2t}, \ldots, \gamma_{7t})'$ by $\gamma_t \equiv G_7 \alpha_t$, and note then that $G_7' \gamma_t = \alpha_t$. We have,

$$(1-B)\gamma_t = (1-B)G_7\alpha_t = G_7\varepsilon_t \equiv \xi_t. \qquad (12)$$

From (12),

$$Var\left(\xi_t\right) = G_7 Var(\varepsilon_t) G_7' = \sigma_\varepsilon^2 G_7 G_7' = \sigma_\varepsilon^2 I_7.$$

Thus the vectors ξ_t have the same scalar variance-covariance matrix as ε_t.

We recover β_t from γ_t in the following manner. Define the 6×7 partitioned matrix H by $H \equiv \left[I_6 \quad \mathbf{0}_6\right] - (1/7)\mathbf{1}_6\mathbf{1}_7'$. Then, if $\alpha_t \equiv \left(\alpha_{1t}, \ldots, \alpha_{7t}\right)'$,

$$\beta_t = H\alpha_t = HG_7'\gamma_t = \Gamma\gamma_t \qquad (13)$$

where $\Gamma \equiv HG_7'$ is the 6×7 matrix given by

$$\Gamma = \left(\tilde{C} \quad \mathbf{0_6}\right), \qquad (14)$$

with $\tilde{C}$ being the 6×6 matrix obtained by taking the first six rows of the matrix C'. From (13) and (14) we obtain β_t as a linear combination of the six coefficients $\tilde{\gamma}_t \equiv \left(\gamma_{1t}, \gamma_{2t}, \ldots, \gamma_{6t}\right)'$ using the equation

$$\beta_t = \tilde{C}\tilde{\gamma}_t. \qquad (15)$$

Substituting (15) into (5) gives

$$TD_t = T_t'\beta_t = T_t'\tilde{C}\tilde{\gamma}_t = (\tilde{C}'T_t)'\tilde{\gamma}_t.$$

This shows that by transforming the trading-day variables each month from T_t to $r_t = \tilde{C}'T_t$, we can write $TD_t = r_t'\tilde{\gamma}_t$ in terms of the vector $\tilde{\gamma}_t$ that has components that follow six independent random walks with common variances σ_ε^2. Having estimated $\gamma_{1t}, \gamma_{2t}, \ldots, \gamma_{6t}$ by signal extraction with the fitted model, we convert these results to estimates of β_t using (15). That is, letting $y \equiv (y_1, y_2, \ldots, y_n)'$ denote the available data, $E(\beta_t \,|\, y) = \tilde{C}E(\tilde{\gamma}_t \,|\, y)$, with signal extraction variance-covariance matrix $Var(\beta_t \,|\, y) = \tilde{C}\, Var(\tilde{\gamma}_t \,|\, y)\, \tilde{C}'$, where $E(\tilde{\gamma}_t \,|\, y)$ and $Var(\tilde{\gamma}_t \,|\, y)$ are respectively the signal extraction mean and covariance matrix of $\tilde{\gamma}_t$. (These were computed here by the REGCMPNT program, though a modification was needed to print the full covariance matrices. Ordinarily the program only prints out the diagonal elements, which are the component signal extraction variances $Var(\gamma_{it} \,|\, y)$.)

4. RESULTS FOR EXAMPLE SERIES

To investigate possible time variation of trading-day effects and to compare results using the alternative models discussed in Section 2, we used REGCMPNT to fit models to three U.S. retail sales time series published by the Census Bureau: sales of department stores (excluding leased departments), sales of women's clothing stores, and sales of shoe stores. All three time series are for the 384-month period from January 1967 to December 1998. (Note: results presented here for department store sales differ some from those presented in Bell (2004) because the time frame of the series used is different, and because benchmark revisions are regularly made to the data over time.)

Let y_t denote the monthly series obtained by taking logarithms of the specific retail sales series in question, after dividing by length of month. In the model for y_t, in addition to trading-day effects, we include an Easter effect with a 10-day window (BH) and an airline model for the remainder:

$$y_t = TD_t + \omega E_t + Z_t,$$

$$(1-B)(1-B^{12})Z_t = (1-\theta B)(1-\Theta B^{12})a_t, \qquad (16)$$

where E_t is the Easter effect variable and ω the associated parameter, and a_t is white noise with variance σ_a^2. We fit (16) with TD_t representing either fixed or stochastically time-varying trading-day effects, using the models discussed in the last two sections.

Figures 1 and 2 are plots of the signal extraction estimates of the time-varying trading-day coefficients β_{it}, $i = 1, 2, \ldots, 7$ over the 384 months for retail sales of department stores and of women's clothing stores, respectively. The estimates for shoe store sales are not shown due to space limitations. The estimates $\hat{\beta}_i$ assuming fixed trading-day effects (3) are included on the plots as a dashed straight line for comparison purposes. Eight different plots of estimated stochastically time-varying coefficients are included on each graph. Seven of these come from Bell's model (6), with each of the days of the week serving as the reference day in one of them. The plot for which the reference day corresponds to the day of the estimated coefficient is plotted with long dark dashes, to differentiate it from the plots for the other six reference days that are plotted as dotted lines. These six plotted curves tend to look similar, and in many cases, some of them approximately coincide, so that the plots for less than six of the reference days are visible. The eighth line in each graph, corresponding to time-varying coefficient estimates from Harvey's model (9), is plotted as a solid line.

For each of the three series, the fixed-effects estimates indicate large positive effects for Friday and Saturday, and a large negative effect for Sunday. Most of the estimated fixed effects for the other days were not significantly different from zero.

For department store sales, the most noticeable feature in the graphs of the time-varying coefficients is the large increase in the effect of Sunday, from an effect on sales of less than -2% early in the series, to a nearly neutral effect at the series' end. Also, while the estimated fixed-effect coefficient for Tuesday is close to zero, the estimated time-varying coefficients for Tuesday show decreases in the effect through time from positive to negative values. The Friday coefficient estimates also decrease over time. Plots of the coefficient estimates for the other days of the week vary less over time.

For retail sales of women's clothing stores, the increase in the estimated time-varying coefficient for Sunday was much less than for the department stores series: a little more than half a percent over the length of the series. The Friday coefficient displayed a similarly moderate decrease, but the estimated coefficients for the other days showed very little movement.

The estimated time-varying coefficients for the data on shoe store sales display more variation than for the series on sales of women's clothing stores. However, the estimated coefficients do not move up or down in a consistent fashion.

Many of the plots of the coefficient estimates from Bell's model for the reference days used in constructing trading-day variables show somewhat erratic variation over time. This highlights the point made earlier, that for Bell's model the coefficient corresponding to the reference day has different statistical properties than the coefficients for the other six days.

For department stores and women's clothing stores, the estimates from Harvey's model behave similarly to the estimates from Bell's model with reference days different from the coefficient day. For the data on retail sales of shoe stores, however, estimating Harvey's model yielded $\hat{\sigma}_\varepsilon^2 \cong 0$. The resulting estimates of the β_{it} had virtually no variation, and in fact nearly coincided with the fixed-effects estimates. In contrast, the estimates from Bell's model vary over time for some of the days, as noted above.

5. CONCLUSIONS AND FUTURE RESEARCH

In this paper we considered the modeling of time-varying trading-day effects using models given in Harvey (1989) and Bell (2004). The models are special cases of RegComponent models (Bell 2004), though in the case of Harvey's model a linear transformation was needed in order to write the model in RegComponent form. The REGCMPNT program was used to fit the models to time series of retail sales of U.S. department stores, women's clothing stores, and shoe stores for the period January 1967 to December 1998.

Estimates from the various models used were mostly in agreement with regard to estimated time variation of the trading-day coefficients for the example series. The main exception was that in Bell's model, estimates of the trading-day coefficient for the reference day used in constructing the trading-day variables often showed erratic time variation. Results thus varied with the choice of the reference day. Though this erratic behavior was not present in all cases and was not necessarily important relative to the overall time variation, it is nonetheless a limitation to Bell's model. The other exception was that for shoe store sales, results from Harvey's model showed no time variation in the coefficients since the one innovation variance for the trading-day component was estimated to be approximately zero. The trading-day coefficients for this example estimated from Bell's models did not move in a consistent upward or downward direction, raising some concerns about whether this estimated time variation is real. On the other hand, since Harvey's trading-day model uses only one variance, if estimation error led to this variance being incorrectly estimated at its boundary value of zero, this would incorrectly suggest no time variation in any of the trading-day coefficients.

The results suggest several topics for future research. One is to examine methods of testing for the presence of time-varying trading-day coefficients. DSQ and DQ considered this topic, but the methods suggested have significant limitations. The work of Harvey and Streibel (1998) on testing for deterministic versus indeterministic cycles may be applicable. Another topic involves generalizing Harvey's model to allow for different innovation variances for the different days. We can easily do this by dropping the common variance assumption made for the random walk models given in (9), and following through with the model derivation as in Section 2. However, if we convert the resulting model to RegComponent form as in Subsection 3.2, the transformation matrix G_7 that will orthogonalize the ε_{it} via (12) will depend on the unknown variances. This generalization cannot be accommodated by our current software.

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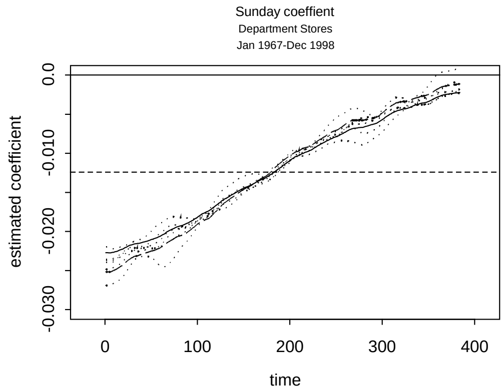
Figure Captions

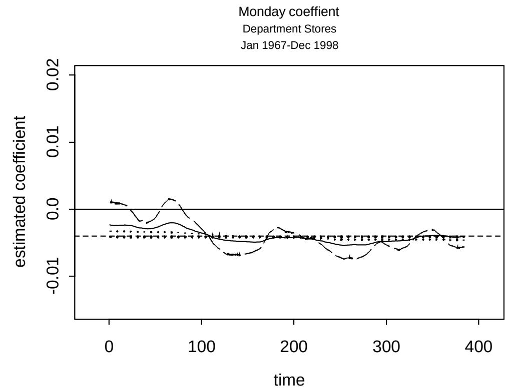
Figure 1. Estimated coefficients of the daily trading-day effects for retail sales in U.S. Department stores (excluding leased departments) during the period January 1967 to December 1998

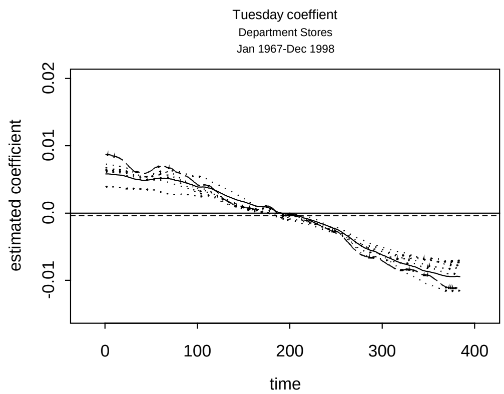
Figure 2. Estimated coefficients of the daily trading-day effects for retail sales in U.S. Women's clothing stores during the period January 1967 to December 1998.

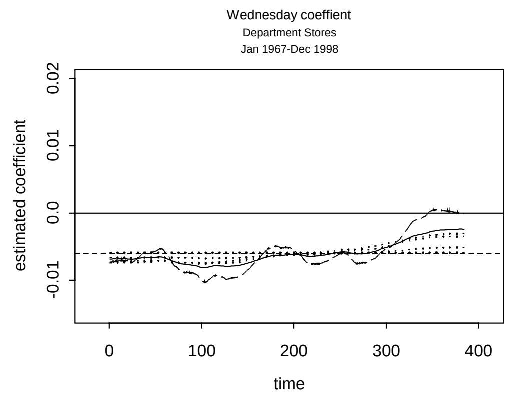
Legend for Figures

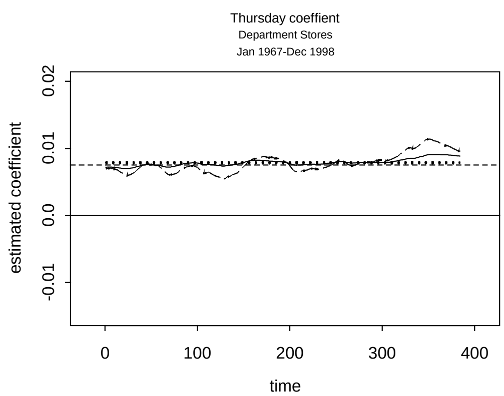
The estimates of the stochastically time-varying coefficient using Bell's model are plotted with a dark dashed line when the reference day corresponds to the day of the parameter estimate, and as dotted lines for other reference days. Estimates from Harvey's model are plotted as a solid line, and the estimated coefficient for the fixed-effect model appears as a dashed straight line.

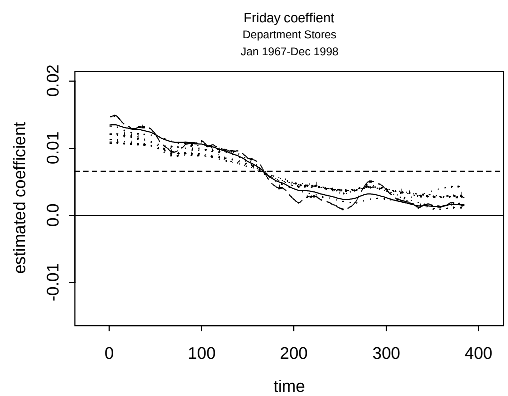

Sunday coeffient
Department Stores
Jan 1967-Dec 1998
estimated coefficient
0.0
-0.010
-0.020
-0.030
0
100
200
300
400
time

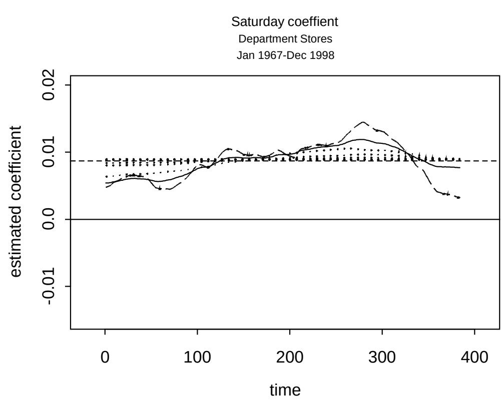

Monday coeffient
Department Stores
Jan 1967-Dec 1998
estimated coefficient
0.02
0.01
0.0
-0.01
0
100
200
300
400
time

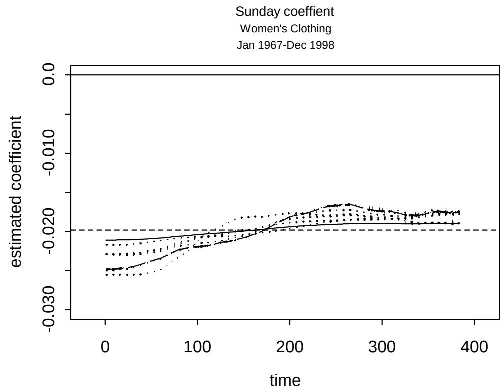

Tuesday coeffient
Department Stores
Jan 1967-Dec 1998
estimated coefficient
0.02
0.01
0.0
-0.01
0
100
200
300
400
time

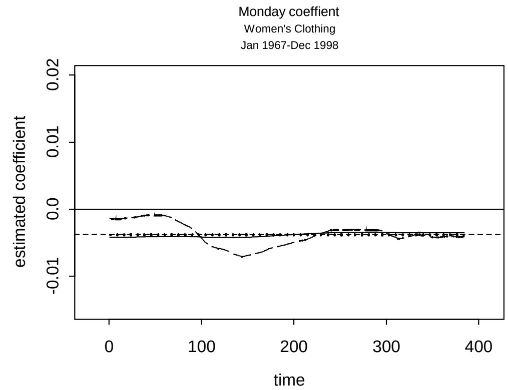

Wednesday coeffient
Department Stores
Jan 1967-Dec 1998
estimated coefficient
0.02
0.01
0.0
-0.01
0
100
200
300
400
time

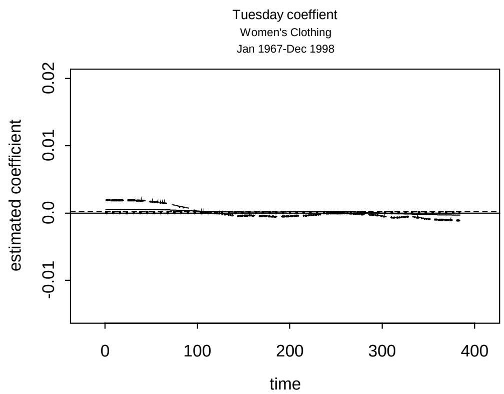

Thursday coeffient
Department Stores
Jan 1967-Dec 1998
estimated coefficient
0.02
0.01
0.0
-0.01
0
100
200
300
400
time

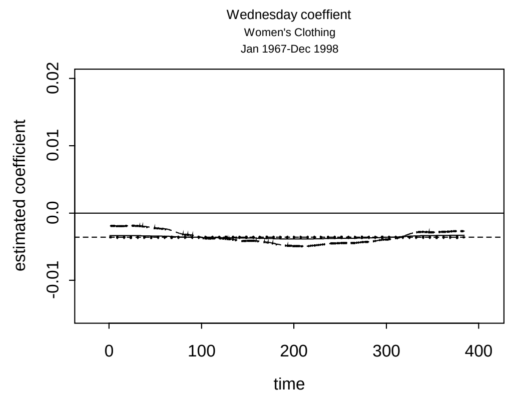

Friday coeffient
Department Stores
Jan 1967-Dec 1998
estimated coefficient
0.02
0.01
0.0
-0.01
0
100
200
300
400
time

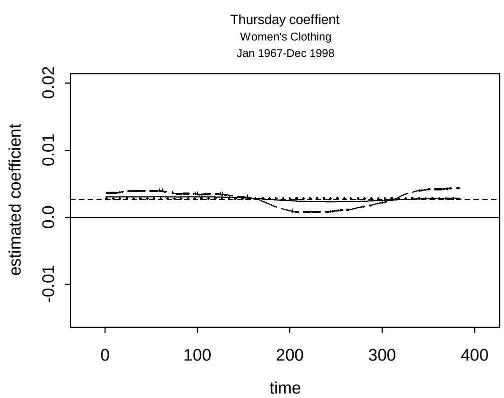

Saturday coeffient
Department Stores
Jan 1967-Dec 1998
estimated coefficient
0.02
0.01
0.0
-0.01
0
100
200
300
400
time

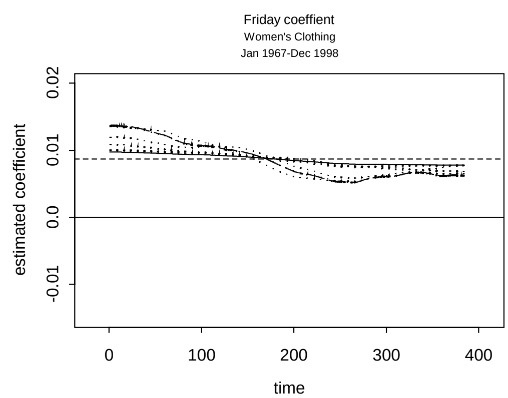

Sunday coeffient
Women's Clothing
Jan 1967-Dec 1998
estimated coefficient
0.0
-0.010
-0.020
-0.030
0
100
200
300
400
time

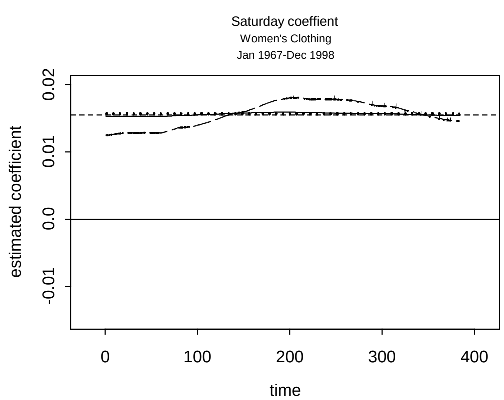

Monday coeffient
Women's Clothing
Jan 1967-Dec 1998
estimated coefficient
0.02
0.01
0.0
-0.01
0
100
200
300
400
time


Tuesday coeffient
Women's Clothing
Jan 1967-Dec 1998
estimated coefficient
0.02
0.01
0.0
-0.01
0
100
200
300
400
time


Wednesday coeffient
Women's Clothing
Jan 1967-Dec 1998
estimated coefficient
0.02
0.01
0.0
-0.01
0
100
200
300
400
time


Thursday coeffient
Women's Clothing
Jan 1967-Dec 1998
estimated coefficient
0.02
0.01
0.0
-0.01
0
100
200
300
400
time


Friday coeffient
Women's Clothing
Jan 1967-Dec 1998
estimated coefficient
0.02
0.01
0.0
-0.01
0
100
200
300
400
time


Saturday coeffient
Women's Clothing
Jan 1967-Dec 1998
estimated coefficient
0.02
0.01
0.0
-0.01
0
100
200
300
400
time