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ESTIMATING TRADING-DAY VARIATION IN MONTHLY ECONOMIC TIME SERIES

by
Allan Young
Economic Research and Analysis Division

U.S. DEPARTMENT OF COMMERCE
John T. Connor, Secretary

BUREAU OF THE CENSUS
A. Ross Eckler, Acting Director

BUREAU OF THE CENSUS
A. Ross Eckler, Acting Director

MORRIS H. HANSEN, Assistant Director for Research and Development

Economic Research and Analysis Division
JULIUS SHISKIN, Chief

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Preface

Many types of monthly economic time series contain variations which are related to the number of times a particular day or days of the week occur in the calendar month. These variations are usually referred to as trading-day variations. Recently, substantial use and development of a technique to estimate and remove these variations have occurred at the Bureau of the Census. This new technique estimates trading-day variation by relating the monthly volume of activity to the number of times each day of the week occurs in the month. By and large, this approach yields better trading-day and seasonal adjustments than the often used techniques that rely upon independent, external evidence of the percent of the week's activity that occurs on each day of the week.

This paper attempts to examine fully the subject of trading-day variation. Briefer treatments of the subject are found in "Census Trading-Day Adjustment Method," Business Cycle Developments, May 1964, and in specifications that will be made available for a new version of the Census Method II seasonal adjustment program, designated the X-11 variant, which includes a routine to estimate trading-day variation.

Several people have made substantial contributions to the recent development of trading-day adjustment techniques. The work at the Bureau of the Census was carried on under the supervision and encouragement of Julius Shiskin. John Musgrave made many valuable contributions and prepared the mathematical presentation in appendixes A and B. Gerald Donahoe assisted in much of the development and application of the method. Morton Somer prepared the computer programs. Harry Rosenblatt and Edward Melnick made helpful suggestions. Norman Bakka, Richard Bartlett, and Barry Beckman provided substantial assistance. Geraldine Censky and Marie Wann provided editorial review.

Much of the work draws upon Stephen Marris' earlier work at the Organization for Economic Cooperation and Development. James Nettles and David Staiger of the Federal Reserve Board made helpful suggestions.

Allan Young
Bureau of the Census
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ESTIMATING TRADING-DAY VARIATION IN MONTHLY ECONOMIC TIME SERIES

I. INTRODUCTION

An important source of the month-to-month variation in nany monthly economic time series is trading-day variation.[1] In activities such as production, sales, and shipments in domestic and foreign trade, the monthly rate of activity is related to the number of working or trading days in the month. A familiar example is retail sales, where more sales are made on Fridays and Saturdays than on other days and higher sales are made in months containing five Fridays and/or Saturdays than those with four.

Trading-day variation is systematic and its characteristics relatively stable over several years. Therefore, it is possible to estimate trading-day variation from information contained in the monthly data.

The justification for a trading-day adjustment is that it reduces the month-to-month variation in seasonally adjusted data so that the trend-cycle component is revealed more clearly.[2] This reduction in month-to-month variation is illustrated in the chart, which shows seasonally adjusted U.S. imports, nonagricultural job placements, and sales of department stores with and without an adjustment for trading-day variation.

The importance of trading-day variation relative to other types of month-to-month fluctuations is shown in table 1 for seven series adjusted by the Bureau of the Census. In each series, trading-day variation is several times as large as the monthly variation in the trend-cycle component. For imports, trading-day variation is considerably more important than seasonal variation, and for wholesale sales, these two components are almost equal. In both cases, about half the total variation is accounted for by trading-day variation. Trading-day variation is of prime concern when attempting to assess the underlying cyclical movement over short spans (1 or 2 months). Over longer spans trading-day variation is of less importance, since it frequently reverses direction and does not cumulate as do the seasonal and cyclical movements.

Trading-day variation cannot be estimated in series where the irregular variation is large. There is an upper limit, in economic series, to the amount of variation arising from trading days. For example, the monthly change induced by a 5-day week[3] averages 5.3 percent and the change induced by a 6-day week averages 2.8 percent. When $\bar{I}$ is between 5 and 10 percent, the monthly change of 3 to 5 percent or less in the trading-day variation ceases to be important and its estimation becomes difficult. Likewise, there is less need for a trading-day adjustment since there are large irregular variations which will continue to obscure the trend-cycle even after the trading-day variation has been removed.

Census Bureau series covering a broad spectrum of economic activity show less variation attributable to the calendar than would be expected. For example, assume that a manufacturing plant operates on a 5-day week. We would expect, then, the volume of activity in a January with 23 workdays (4 Saturdays and 4 Sundays) to be 9 percent higher than in a January with 21 workdays (5 Saturdays and 5 Sundays). This often does not appear to be the case. Instead, the January with 23 workdays is only about 4- or 5-percent higher. Such a phenomenon can arise either from factors inherent in the economy or from various practices of recording and reporting data. It appears that a substantial proportion of economic activity occurs on the basis of monthly plans and schedules that are drawn up with little or no attention to the number of trading days within the calendar months and/or is recorded and reported on a basis that takes little account of the number of trading days in calendar months.

The implication of this moderation in trading-day variation is that the narrow definition implicit in many trading-day adjustments does not allow for the types of variation that may actually exist in the monthly data. Using such a definition of trading-day variation, one might, after noting that an economic activity operates on a 5-day week, proceed

[1] The term "trading-day variation" can be considered interchangeable with the terms "working-day variation" and "calendar variation."

[2] A familiarity with seasonal-adjustment techniques is assumed. At times, reference is made to specific measures provided by the Census Method II ratio-to-moving-average method of seasonal adjustment. Details concerning Census Bureau methods can be found in references 7 and 9 (see end of paper).

[3] A set of daily weights where Mon., . . . , Fri. = 1.4; Sat., Sun., = 0.0 is commonly referred to as a 5-day week (also 6-day, 5 1/2 day, etc.).

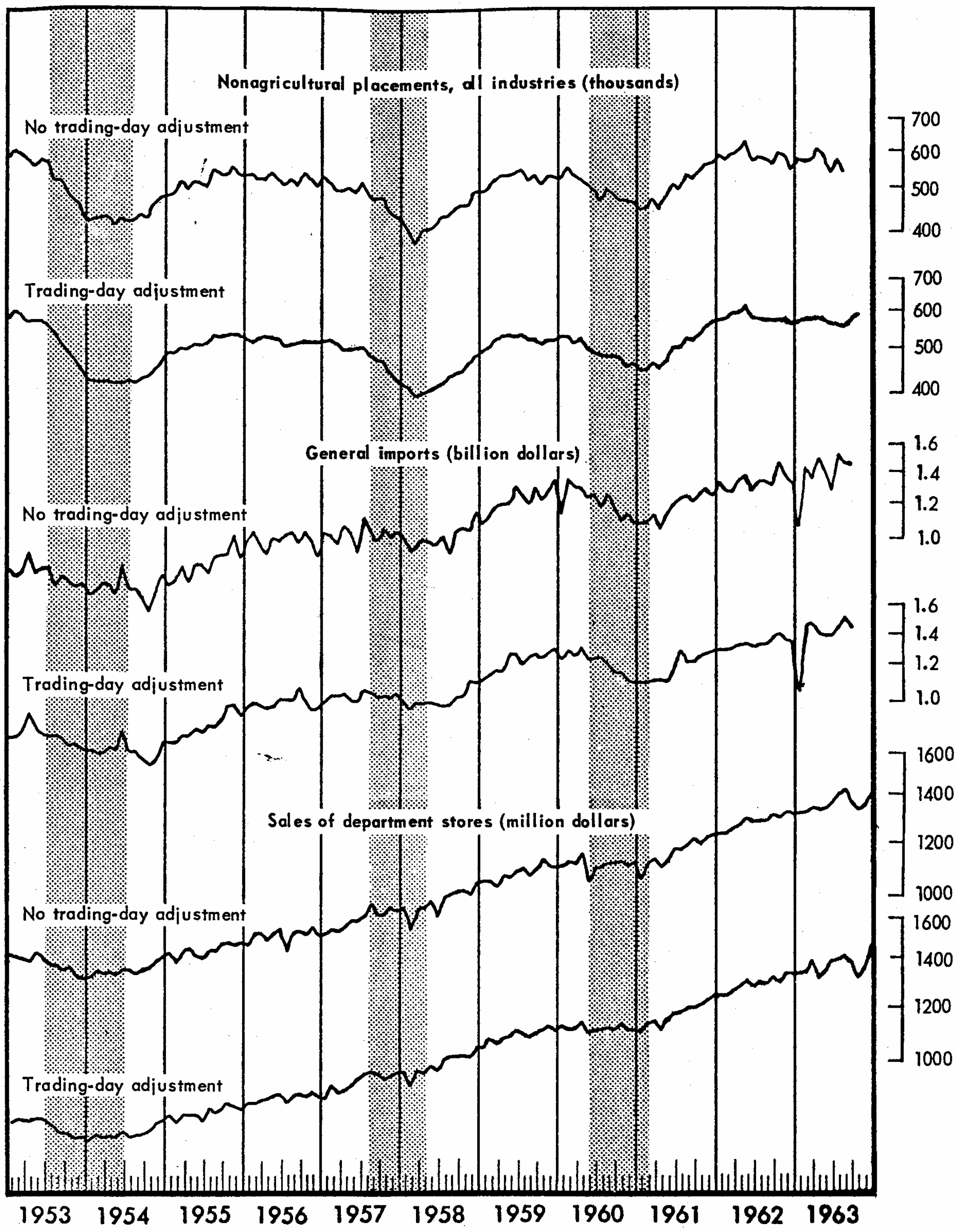
Table 1.--PERCENT DISTRIBUTION OF VARIATION ATTRIBUTABLE TO THE VARIOUS COMPONENTS OF MONTH-TO-MONTH VARIATION, FOR SELECTED BUSINESS ACTIVITIES DURING INDICATED TIME PERIODS

Business activity	Time period	Components of month-to-month variation				
		Trading-Day variation	Seasonal variation	Irregular variation	Trend-cycle variation	Total variation
Retail sales	1953-63	7.7	[1]91.1	0.8	0.4	100.0
Wholesale sales	1960-63	47.8	48.7	2.9	0.6	100.0
Manufacturers' shipments	1953-62	20.1	71.9	5.6	2.5	100.0
Manufacturers' new orders	1953-62	19.3	62.6	14.3	3.8	100.0
U.S. exports	1953-63	18.3	63.1	15.8	2.8	100.0
U.S. imports	1953-63	50.3	38.2	10.0	1.5	100.0
Building permits	1954-62	21.2	69.1	7.8	1.9	100.0

[1] Includes a slight contribution from variation in sales of selected kinds of business in March and April because of shifting date of Easter.

Note: See appendix C for the derivation of these measures from Census Method II summary measures of monthly change, $\bar{O}$, $\bar{S}$, $\bar{I}$, $\bar{C}$.

COMPARISON OF SELECTED ECONOMIC SERIES BEFORE AND AFTER TRADING-DAY ADJUSTMENT



The shaded areas represent contraction periods in general business activity.

Sources of data: Nonagricultural placements, all industries--Department of Labor, Bureau of Employment Security (seasonal and trading-day adjustment by Bureau of the Census); general imports, total, and sales of department stores--Bureau of the Census.

Estimated daily weights used in making the trading-day adjustments:

	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.	Sun.
Nonagricultural placements....................	1.00	1.38	1.26	.97	1.23	.45	.71
General imports...........................	1.61	1.47	1.33	1.33	1.26	0.00	0.00
Department store sales....................	.94	1.00	.99	1.14	1.17	1.35	.42

to adjust for a 5-day week with no activity on Saturday and Sunday. That is, the series would be placed on a daily-rate basis by dividing by the number of weekdays in each month. Results presented below show that such a procedure often yields much poorer results than do the techniques described in this paper which relate the variations in the monthly series to the number of times that each day of the week occurs in the month.

This paper describes the concept of trading-day variation. It reports the techniques for estimating trading-day variation and the results of applying these methods to artificial and real economic series. It also presents tests for determining if trading-day variation is present. On the basis of the work reported in this paper, a routine to estimate trading-day variation is being added to the Census Method II seasonal adjustment program.

II. DEFINITIONS

A. Characteristics of the Calendar

Before defining trading-day variation in economic series, an understanding of two features of the calendar is necessary. We are familiar with the fact that months vary in length—31 days, 30 days, and Februaries of 28 and 29 days. Such variation between 31-day months, 30-day months, and Februaries is termed "length-of-month" or "between-month" variation.

In addition to this variation, the composition of each calendar month varies from year to year. Twenty-two types of calendar months occur—seven 31-day months, seven 30-day months, and eight Februaries.[4] Let us examine them by the day on which they begin. Thirty-one-day months beginning on Monday contain five Mondays, Tuesdays, and Wednesdays and four of the other days of the week. Thirty-one-day months beginning on Tuesday contain five Tuesdays, Wednesdays, and Thursdays and four of the other days of the week, and so on for the other types of 31-day months. (See table 2). In a similar fashion, 30-day months beginning on Monday contain a fifth Monday and Tuesday and so on. Februaries contain 28 days except in leap years. There are seven types of leap-year Februaries distinguished by the beginning day of the month. This variation between months of equal length in the number of times a particular day or days of the week occur in a calendar month is termed "calendar composition" or "within-month variation."

A 28-year cycle occurs before the calendar begins to repeat the same pattern of days and months. (The 28-year cycle is broken at the beginning of each century which is not divisible by 400; i.e., at 1900 and 2100, but not at 2000.) Dates of movable holidays such as Easter do not conform to a regular pattern during the 28-year cycle. A calendar of these 22 types of months for the 28-year period, 1944-71, is shown as table 3. By subtracting or adding 28 years to the period shown, the calendar can be adapted for the periods 1916-43 and 1972-99.

Let us look at these properties of the calendar in more detail by examining a tabulation of the number of weekdays, Saturdays, and Sundays for 5 years, 1960-64, and the average number over the 28-year cycle shown in table 4, page 4. Note the magnitude of the within-month variation. May, a 31-day month, varies from 21 to 23 weekdays a month, a 9-percent range; while June, a 30-day month varies from 20 to 22 week-

[4] If month-to-month changes are considered there are 49 combinations in the calendar. There are 7 combinations between each of the sequences of 31- to 30-day, 30- to 31-day and 31- to 31-day months and 14 between 31-day months to Februaries and Februaries to 31-day months. This paper considers the pattern for 22-month types appropriate for series such as production and sales. For series such as changes in inventories it may be useful to consider the 49 combinations of monthly changes.

Table 2.--NUMBER OF TIMES EACH DAY OF THE WEEK OCCURS IN EACH OF 22 TYPES OF MONTHS

Type-of-month code	First day of month	Number of specified days per month: Sundays	Mondays	Tuesdays	Wednesdays	Thursdays	Fridays	Saturdays
		31-day months						
1	Monday.......	4	5	5	5	4	4	4
2	Tuesday......	4	4	5	5	5	4	4
3	Wednesday....	4	4	4	5	5	5	4
4	Thursday.....	4	4	4	4	5	5	5
5	Friday.......	5	4	4	4	4	5	5
6	Saturday.....	5	5	4	4	4	4	5
7	Sunday.......	5	5	5	4	4	4	4
		30-day months						
8	Monday.......	4	5	5	4	4	4	4
9	Tuesday......	4	4	5	5	4	4	4
10	Wednesday....	4	4	4	5	5	4	4
11	Thursday.....	4	4	4	4	5	5	4
12	Friday.......	4	4	4	4	4	5	5
13	Saturday.....	5	4	4	4	4	4	5
14	Sunday.......	5	5	4	4	4	4	4
		Leap-year Februaries						
15	Monday.......	4	5	4	4	4	4	4
16	Tuesday......	4	4	5	4	4	4	4
17	Wednesday....	4	4	4	5	4	4	4
18	Thursday.....	4	4	4	4	5	4	4
19	Friday.......	4	4	4	4	4	5	4
20	Saturday.....	4	4	4	4	4	4	5
21	Sunday.......	5	4	4	4	4	4	4
		Non-leap-year Februaries						
22	Any day......	4	4	4	4	4	4	4

Table 3.—CALENDAR, BY TYPE-OF-MONTH CODE, 1944 to 1971

(Figures are type-of-month codes. See table 2 for the number of times each day of the week occurs in each of the months)

Year	January	February	March	April	May	June	July	August	September	October	November	December
1944......	6	16	3	13	1	11	6	2	12	7	10	5
1945......	1	22	4	14	2	12	7	3	13	1	11	6
1946......	2	22	5	8	3	13	1	4	14	2	12	7
1947......	3	22	6	9	4	14	2	5	8	3	13	1
1948......	4	21	1	11	6	9	4	7	10	5	8	3
1949......	6	22	2	12	7	10	5	1	11	6	9	4
1950......	7	22	3	13	1	11	6	2	12	7	10	5
1951......	1	22	4	14	2	12	7	3	13	1	11	6
1952......	2	19	6	9	4	14	2	5	8	3	13	1
1953......	4	22	7	10	5	8	3	6	9	4	14	2
1954......	5	22	1	11	6	9	4	7	10	5	8	3
1955......	6	22	2	12	7	10	5	1	11	6	9	4
1956......	7	17	4	14	2	12	7	3	13	1	11	6
1957......	2	22	5	8	3	13	1	4	14	2	12	7
1958......	3	22	6	9	4	14	2	5	8	3	13	1
1959......	4	22	7	10	5	8	3	6	9	4	14	2
1960......	5	15	2	12	7	10	5	1	11	6	9	4
1961......	7	22	3	13	1	11	6	2	12	7	10	5
1962......	1	22	4	14	2	12	7	3	13	1	11	6
1963......	2	22	5	8	3	13	1	4	14	2	12	7
1964......	3	20	7	10	5	8	3	6	9	4	14	2
1965......	5	22	1	11	6	9	4	7	10	5	8	3
1966......	6	22	2	12	7	10	5	1	11	6	9	4
1967......	7	22	3	13	1	11	6	2	12	7	10	5
1968......	1	18	5	8	3	13	1	4	14	2	12	7
1969......	3	22	6	9	4	14	2	5	8	3	13	1
1970......	4	22	7	10	5	8	3	6	9	4	14	2
1971......	5	22	1	11	6	9	4	7	10	5	8	3

NOTE: By subtracting or adding multiples of 28 years, the calendar can be used for the period 1901 to 2099. The 28-year cycle is broken at 1900 and 2100 because these years are not leap years. For 1900 and earlier years, see an almanac or a perpetual calendar.

days a month, a 10-percent range. Such variation arises only when we distinguish between particular days of the week.

Also note that the 3 percent difference in length of month between May with 31 days and June with 30 days appears in the average number of weekdays over the 28-year period, 22.14 weekdays for May and 21.43 for June. Unlike within-month, length-of-month variation arises whether we consider the total number of days or distinguish between particular days of the week in the month.

B. Definition of Trading-Day Variation

Trading-day variation could be described as the variation in a monthly economic time series related to the calendar composition and length-of-month variation in the calendar. However, variations in the volume of activity arising from months of different length (except for leap-year and non-leap-year Februaries) cannot be statistically separated from other seasonal influences also causing differences between months. Therefore, length-of-month variation is defined and estimated as part of the seasonal component.[5] On the other hand, within-month variation which varies abruptly from year to year within a month, is not included in the definition or estimation of the seasonal. Our definition, then, becomes simply this: Trading-day variation is the monthly variation in a series related to the within-month variation or calendar composition.

[5]The seasonal can be defined as a constant or gradually increasing or decreasing function within a month over several years.

Table 4.—NUMBER OF WEEKDAYS, SATURDAYS, AND SUNDAYS IN MAY AND JUNE 1960 to 1964

Year	May			June		
	Week-days	Satur-days	Sun-days	Week-days	Satur-days	Sun-days
1960.........	22	4	5	22	4	4
1961.........	23	4	4	22	4	4
1962.........	23	4	4	21	5	4
1963.........	23	4	4	20	5	5
1964.........	21	5	5	22	4	4
28-year average.....	22.14	4.43	4.43	21.43	4.29	4.29

C. Internal vs. External Evidence

Note that the definition of trading-day variation, above, is in terms of the monthly variation in the series. The trading-day adjustment implied by this definition is derived from the internal evidence contained in the monthly data. This is a more general approach than that of the frequently used techniques which rely upon external or a priori evidence of the daily rates of activity in making a trading-day adjustment. The differences between the use of internal and external evidence are subtle and are considered in some detail in this section.

The customary use of external evidence is to estimate the proportion of the week's activity occurring, on average, on each day of the week. In the remainder of this paper these estimates are referred to as the seven actual daily rates of activity. The available external evidence often consists of the daily schedule of hours of work, direct personal observation of the activity, or records of the daily volume of activity. Although such evidence is usually limited, it is often considered sufficient to provide estimates of seven actual daily rates of activity.

To make the trading-day adjustment, each monthly figure is divided by a factor constructed by aggregating the daily rates to a monthly sum. The construction of the factors can be represented as follows:

(Equation 1)

$$M_i = \frac{X_{1i}r_1 + X_{2i}r_2 + \ldots + X_{7i}\, r_7}{N_i} \quad (i = 1, \ldots, n),$$

where M_i is the trading-day adjustment factor for month i;

X_{ji} is the number of times day-of-the-week j occurs in month i;

r_j is the daily rate, i.e., the proportion of the week's activity that occurs on the day-of-the-week j ($j = 1, \ldots, 7$), where $\sum_{1}^{7} r_j = 7$;

N_i is either 31, 30, or 28.25 depending upon whether month i is a 31-day month, 30-day month, or February;

n is the number of months of data available.

Use of such factors implies that the seven actual daily rates (the r's) can be aggregated to weekly and then to monthly levels corresponding to the actual monthly variation.

This assumption implied in the use of seven actual daily rates of activity is incorrect when part or all of the economic process operates under schedules that do not take into account the calendar composition of the month. Allowance must be made for the relation of the activity on each day of the week to the monthly volume of activity rather than for the relation of the daily activity to the weekly volume of activity. The relation of the activity on each day of the week to the monthly volume can vary with the calendar composition of the month, while the actual daily rates that are proportions of the weekly volume remain fixed.

The following simplified example illustrates the relationship that sometimes tends to exist between the daily, weekly, and monthly activity. Suppose that all consumers receive one-twelfth of their annual income each month and that they dispose of all their month's income before the next payday by shopping at retail stores. If the retail stores are open Monday through Saturday, the seven daily rates of activity include a zero Sunday, and we can assume the other days to be equal; i.e., Mon., . . ., Sat. = 1.17; Sun. = 0.0; Total = 7.00. On the basis of the daily rates of activity, we are led to conclude that the monthly volume of sales in a January with 23 shopping days is 9 percent higher than in a January with 21 shopping days, other things being equal. However, this is not the case. Since the consumers dispose of an equal amount of income in both Januaries, the monthly volume of sales in the two Januaries is the same. Although the daily rates expressed as a percent of the week remain the same in the two months, the daily activity for Monday through Saturday expressed as a percent of the total monthly activity is higher by 9 percent when the monthly total is spread over 21 rather than 23 shopping days. In this example, the occurrence of Sunday or any other day five rather than four times has no effect upon the monthly variation and knowledge of the daily activity as a percent of the week's activity is extraneous.

The same relationship could be illustrated if, instead of assuming a fixed amount of income each month, we assume a continuous daily or weekly use that affects the timing of the purchases. This possibility is most obvious for grocery sales. The amount of groceries purchased by the consumer in January might reflect the fact that approximately the same amount of food is placed upon his dining table regardless of the calendar composition of the month.

Note that in one respect the two examples are different. In the first, the fixed amount of activity that occurs each January, regardless of calendar composition, is the same as the fixed amount that would occur in a 30-day month or a February. In the second example the amount of activity that is fixed with respect to calendar composition is not fixed with respect to the length of the month. More food is used in 31-day months than in 30-day months or in Februaries. At times, recognition of these two possibilities will be useful.

There may also be other types of relationships between daily, weekly, and monthly activity that are more complex than that implied in the use of seven actual daily rates. For example, total activity this month might reflect next month's demand which varies with next month's calendar composition. To allow for such complex variations, the relations of the daily activity to the monthly activity must be examined for each type of calendar composition. This examination can be done either by compiling much more external evidence of daily activity than is customary or practical, or by statistically relating calendar composition to the variation in the monthly activity. Although the above examples are oversimplified and extreme, it appears reasonable to assume that there are, in many areas of the economy, tendencies towards more complex relationships than those implied by the use of seven daily rates of activity. The empirical evidence presented in section IV, B supports such a supposition. Even though there may be other unknown explanations for the empirical evidence, it appears worthwhile for the present to accept the above line of reasoning rather than the rigid approach imposed by the use of seven actual daily rates of activity.

A second problem with external evidence is the availability of information. Often the cost of obtaining the information forces the estimation of the actual daily rates to be based upon insufficient information, particularly with respect to such factors as overtime practices and continuous process activities that operate around the clock. The availability of information is a problem, whether or not the assumption implied by the use of seven actual daily rates of activity holds true for a particular series.

Finally, there is a third problem which is an extension of the second. Often there are unobserved bookkeeping, reporting, and data-processing practices related to calendar composition that are at work modifying the actual variations. These modifications cannot readily be observed from external evidence.

The first and third shortcomings of external evidence discussed above are not present when an adjustment is derived from internal evidence. An adjustment based upon internal evidence allows for the total variation that can be related to calendar composition; that is, to the occurrence of a particular day of the week five times rather than four times in a

Table 5.--IRREGULAR COMPONENT FOR DEPARTMENT STORE SALES, ARRANGED BY CALENDAR COMPOSITION, 1953 to 1962

(I = Irregular component; date = month and year of basic data)

31-day months beginning on--													
Sunday		Monday		Tuesday		Wednesday		Thursday		Friday		Saturday	
I	Date	I	Date	I	Date	I	Date	I	Date	I	Date	I	Date
99.2	Mar. 1953	100.4	Mar. 1954	100.8	Dec. 1953	101.0	July 1953	101.0	Jan. 1953	100.5	May 1953	98.5	Aug. 1953
97.9	Aug. 1954	99.3	Aug. 1955	101.2	Mar. 1955	101.1	Dec. 1954	101.1	Oct. 1953	98.1	Jan. 1954	98.0	May 1954
98.2	May 1955	99.4	Oct. 1956	101.3	May 1956	101.6	Aug. 1956	101.2	July 1954	98.6	Oct. 1954	100.7	Jan. 1955
98.1	Jan. 1956	100.1	July 1957	99.8	Jan. 1957	101.2	May 1957	99.6	Dec. 1955	100.6	July 1955	100.5	Oct. 1955
95.3	July 1956	101.4	Dec. 1958	98.4	Oct. 1957	103.0	Jan. 1958	100.9	Mar. 1956	98.7	Mar. 1957	97.8	Dec. 1956
100.1	Dec. 1957	100.3	Aug. 1960	101.5	July 1958	100.4	Oct. 1958	103.9	Aug. 1957	101.1	Aug. 1958	99.9	Mar. 1958
95.8	Mar. 1959	100.2	May 1961	99.2	Dec. 1959	101.2	July 1959	103.3	May 1958	100.4	May 1959	97.3	Aug. 1959
96.1	May 1960	100.4	Jan. 1962	99.1	Mar. 1960	101.9	Mar. 1961	102.1	Jan. 1959	99.9	Jan. 1960	101.2	Oct. 1960
96.5	Jan. 1961	98.3	Oct. 1962	100.9	Aug. 1961	100.8	Aug. 1962	102.8	Oct. 1959	99.9	July 1960	99.5	July 1961
97.7	Oct. 1961	.....	....	102.5	May 1962	.....	....	101.7	Dec. 1960	101.3	Dec. 1961	96.1	Dec. 1962
99.8	July 1962	.....	....	.....	....	.....	....	101.2	Mar. 1962	.....	....	.....	....
Mean													
97.7		100.0		100.5		101.4		101.7		99.9		99.0	

30-day months beginning on--													
Sunday		Monday		Tuesday		Wednesday		Thursday		Friday		Saturday	
I	Date	I	Date	I	Date	I	Date	I	Date	I	Date	I	Date
98.9	Nov. 1953	100.6	June 1953	99.8	Sept.1953	100.2	Apr. 1953	101.4	Apr. 1954	102.8	Apr. 1955	100.5	Sept. 1956
98.3	Apr. 1956	101.1	Nov. 1954	101.4	June 1954	100.2	Sept.1954	101.2	Sept.1955	102.3	June 1956	98.9	June 1957
97.9	Sept.1957	99.2	Apr. 1957	100.1	Nov. 1955	98.3	June 1955	102.7	Nov. 1956	101.2	Nov. 1957	97.1	Nov. 1958
96.0	June 1958	99.8	Sept.1958	100.3	Apr. 1958	101.3	Apr. 1959	100.1	Sept.1960	104.3	Apr. 1960	97.4	Apr. 1961
98.6	Nov. 1959	101.1	June 1959	99.7	Sept.1959	99.8	June 1960	101.3	June 1961	100.6	Sept.1961	99.6	Sept. 1962
98.2	Apr. 1962	.....	....	99.5	Nov. 1960	100.6	Nov. 1961	102.3	Nov. 1962	99.2	June 1962	.....	....
Mean													
98.0		100.4		100.1		100.1		101.5		101.7		98.7	

Februaries (Mean 99.0[1])																			
I	Year	I	Year	I	Year	I	Year	I	Year	I	Year	I	Year	I	Year	I	Year	I	Year
99.7	1953	100.6	1954	97.1	1955	[2]101.5	1956	100.8	1957	95.1	1958	100.5	1959	[2]100.7	1960	100.9	1961	97.2	1962

[1]Does not include leap-year Februaries. [2]Leap year.

month.[6] The second shortcoming, lack of sufficient information, is sometimes a problem with internal evidence in the sense that statistically reliable estimates cannot be obtained when there are too few observations or there is too large an unexplained residual.

III. ESTIMATION

In the standard ratio-to-moving-average methods of seasonal adjustment such as Census Method II, the unadjusted data is sequentially separated into three components, trend-cycle, seasonal, and irregular (i.e., the residual). When trading-day variation is present in the unadjusted data, it is primarily included in the estimate of the irregular component, rather than the seasonal or trend-cycle, since in many ways it more closely resembles the random fluctuations in the irregular component. (Over a period of months trading-day variation frequently reverses direction while the trend-cycle is a smooth curve; over a period of years within a month it also frequently reverses direction while the seasonal is a constant or a gradually increasing or decreasing curve.) To estimate trading-day variation from internal evidence, it is necessary, therefore, to examine the estimate of the irregular component (more accurately termed the combined irregular and trading-day-variation component) provided by the ratio-to-moving-average seasonal adjustment.

In simultaneous solutions for the seasonal, trend-cycle, and irregular components an allowance for trading-day variation can be included in a manner analogous to those described below for the ratio-to-moving average method.

A. Grouping Months by Calendar Composition

The simplest approach to estimating trading-day variation is to sort the values of the combined irregular and trading-

[6]Some earlier studies have considered trading-day variations to be more complex than merely counting the number of days the store is open each month: (a) Kuznets (4) states that "To establish the number of working days in an industry is often impossible, the number being at best an estimated. In many economic processes it is difficult to assume that volume of activity is directly proportional to the number of working days (for example bank clearings or retail sales)"; (b) Eisenpress (3) presents a method of adjusting bank debits for trading-day variation where Sunday, a nontrading day (banks are closed), does not receive a zero weight, but one determined by the variation in the monthly series; (c) Marris (5) widens the concept of trading-day variation and introduces the effect of reporting practices upon trading-day variation. Marris also presents a summary of other work in the field. (See references at end of paper.) The present paper attempts to further modify the concept of trading-day variation suggested by Marris by considering in more detail and emphasizing the factors that modify the actual daily rates of activity. These factors virtually require that trading-day variation be considered at the level of monthly activity, not daily activity.

day component on the basis of calendar composition, thus placing them in 22 different groups. Significant variation between these groups is evidence of trading-day variation. For example, the irregular values for months that contain five Saturdays might tend to be higher or lower than the irregular values for months with four Saturdays. To make a trading-day adjustment, the means of the irregular values for each group are computed and then divided into the series.

Table 5 illustrates this technique for making a trading-day adjustment. It shows the irregular component of department store sales arranged by calendar composition. The means of the groups of irregulars, shown at the bottom of the table, vary between 101.7 (for 31-day months beginning on Thursday) and 97.7 (for 31-day months beginning on Sunday). Because of the length of the series, no means are shown for leap-year Februaries.

This technique for adjusting for trading-day variation is similar to the technique of computing stable seasonals where all Januaries are placed together, all Februaries, and each of the other months, and the mean for each group is taken as an estimate of the seasonal. The only difference is the framework in which the data is ordered. To compute the seasonal, we group months of like name because the rate of activity is similar in the same month each year. To compute the trading-day factor, we group together months of like calendar composition, since it is the number of each type of day of the week in the month that gives rise to trading-day variation.

B. Estimating Daily Weights

It is possible to refine the above approach to trading-day variation. The means of the irregular values for each of the 22 groups are only estimates of the effect of particular days of the week in combination with adjacent days. Also, in a series of less than 28 years, there is only one observation for some types of leap-year Februaries and none for the other types. The refinement is to estimate seven weights, one for each day of the week, using the data from all months. By reducing the system to seven estimates, the separate effect of each day of the week is determined and the reliability of the estimates is increased.

These seven weights, at the risk of confusion with the seven actual daily rates of activity discussed before, will be referred to as daily weights. Before proceeding, it is necessary to stress the relation between these seven daily weights derived from internal evidence and the seven daily rates derived from external evidence. The daily weights will correspond to the actual daily rates if the variation in the monthly volume of activity corresponds to the variation in the monthly factors constructed from the daily rates, r_j, in equation 1. Otherwise the daily weights will not correspond with the actual daily rates. When there is no correspondence, either the economic process contains complex relationships between the daily, weekly, and monthly volumes of activity or there are bookkeeping and reporting practices that are affecting the data. Experience at the Census Bureau suggests that often there is sufficient lack of correspondence between the daily rates and the actual daily weights to frustrate attempts to interpret the estimated daily weights in light of a known pattern of daily activity. (See, for example, the estimated daily weights in table 8.) The daily weights must usually be viewed only as statistical weights representing the effect of several variables. In the next section we shall return to the problem of interpreting the daily weights.

The Bureau of the Census has used two methods of estimating these daily weights. The first method was developed at the Organization for Economic Cooperation and Development (OECD)(5). It also was developed and used by the Federal Reserve Board.[7] Its chief merit is that the computations are simple and can be done by hand. The method now used at the Census Bureau and being introduced in the Census Method II seasonal-adjustment program is a regression method which makes more complete use of the data and yields more reliable estimates then the OECD method. Since multiple regression techniques are used, it requires an electronic computer for efficient application. Also, it can be combined more readily with holiday or seasonal adjustments in simultaneous solutions rather than the sequential solution used in Census Method II.

The trading-day routine that includes the regression estimates of the daily weights (developed more fully in appendix A) is being added to Census Method II in the following sequence:

(1) A seasonal adjustment is first made with no trading-day adjustment or with a given set of daily weights (referred to as a prior adjustment). This adjustment produces an irregular component which is composed of (a) the "true" irregular and (b) the trading-day variation (or the residual trading-day variation if a prior adjustment was made).

(2) This irregular is then modified for extreme values which would tend to distort the estimated daily weights and regressed upon seven independent variables representing the number of times each day of the week occurred in the month in the following manner:

(Equation 2)

$$I_i = \frac{X_{1i}B_1 + X_{2i}B_2 + \cdots + X_{7i}B_7 + E_i}{N_i},$$

where I_i is the modified irregular for month i and $E[I_i] = 1$;

X_{ji} is the number of times day-of-the-week j occurs in month i;

B_j's are the seven daily weights, where $\sum_1^7 B_j = 7$;

N_i is either 31, 30, or 28.25 depending upon whether month i is a 31-day, 30-day month, or February;

E_i is the "true" irregular for month i.

(3) The trading-day adjustment factors are the estimated values $\hat{I}_i$ of the I_i, that is,

(Equation 3)

$$\hat{I}_i = \frac{X_{1i}b_1 + X_{2i}b_2 + \ldots + X_{7i}b_7}{N_i},$$

where b_j is the least-squares estimate of B_j ($j = 1, \ldots, 7$).

In the Method II routine, they are divided into the unadjusted data and then a second seasonal adjustment is made, based upon trading-day adjusted data. Details of how the trading-day routine is inserted in the Method II sequence will be included in forthcoming specifications for a new version of the Method II program.

[7]The OECD article (5) presents equations for 31-day months only. Federal Reserve Board and Census Bureau have extended the approach by also making estimates of daily weights from the irregulars for 30-day months and combining the 31- and 30-day estimates by weighting them by the ratios 7/11 and 4/11, respectively. The combined estimates were found to have smaller σ's than the 31-day month estimates.

If a prior trading-day adjustment was made, the estimated set of residual weights (b_j) may be combined with the set of prior weights (p_j) to obtain total weights (D_j) by the formula $D_j = b_j + P_j - 1$.

(4) One may perform a standard t-test to determine whether an estimated weight b_j is significantly different from any specified value and an F-test to determine the significance of the regression (i.e., the existence of significant trading-day variation in the irregular).

C. Interpreting Daily Weights

As stated above, if the assumption implied by the use of seven daily rates of activity holds true for a paticular series, the estimated daily weights will correspond with the actual daily rates of activity. (This can be seen by comparing equations 1 and 2.) For example, the negligible weekend activity in many areas of the economy is reflected in estimated weights approximating Mon., . . . , Fri. = 1.4; Sat., Sun. = 0.0 for some types of series. Another instance is the retail sale of nondurable goods where the daily-sales pattern tends to be reflected in the estimated weights, Mon. = 0.9, Tues. = 0.9, Wed. = 0.9, Thurs. = 1.0, Fri. = 1.4, Sat. = 1.4, Sun. = 0.5.

When none of the monthly activity varies with the calendar composition (there is no trading-day variation), the estimated daily weights all equal "1", regardless of the pattern of activity within the week. When only a portion of the series is independent of calendar composition, the estimated daily weights can be considered as composed of two parts, the first part having differential weights for each day and the other consisting of equal weights for each day. Consider the following hypothetical set of daily weights:

Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.	Total of weights	Percent
1.20	1.20	1.20	1.20	1.20	0.50	0.50	7	100

These weights can be separated into two parts:

A..	.70	.70	.70	.70	.70	0.0	0.0	3.5	50
B..	.50	.50	.50	.50	.50	.50	.50	3.5	50

where "A" reflects that part of the series which varies with the number of different-type days and accounts for 50 percent of the series, and "B" is independent of calendar composition and accounts for 50 percent of the total series. Part B of the weights may be divided into two types of activity: (B1) activity which varies only with the length of the month, and (B2) activity which is constant from month to month.[8] In addition, bookkeeping practices can modify the actual variations represented by the A and B parts of the weights.

While a particular series may not contain all the types of variations mentioned above, such types of variations are widespread. Consider, for example, activity at the manufacturing level. First, in many industries a 5-day week is prevalent, which exerts a tendency for the total monthly activity to vary with the number of weekdays. This activity is represented by "A". Second, there is some overtime work and continuous processes that operate 7 days a week. There may also be monthly schedules or continuing demand which results in activity that is independent of calendar composition and varies with the length of the month. This activity is represented in "B1". Third, there may be some activities such as those operating under contracts calling for one-twelfth of the year's scheduled activity to be completed each month, that are independent of calendar composition and the length of the month, represented in "B2". Finally, bookkeeping practices can be represented in either "A" or "B"

Bookkeeping reports that do not cover precisely the calendar month affect trading-day variation and thereby the estimated daily weights. For example, suppose a firm that operates on a 6-day week, (Mon., . . . , Sat. = 1.17, Sun. = 0.0) follows the practice of closing its books for the month on Friday whenever the month ends on Saturday (which occurs only about twice a year) and including the Saturday activity in the following month. This practice would apply to 31-day months beginning on Thursday and 30-day months beginning on Friday. For such months, reported sales would be decreased by almost 4 percent. Conversely, reported sales are increased by almost 4 percent for the months following, which always begin on Sunday. Reporting sales in this manner results in a monthly series which yields daily weights of Mon., . . . , Fri. = 1.17, Sat. = 0.0 and Sun. = 1.17 rather than the actual daily rates, which had a zero Sunday.[9]

Another bookkeeping practice that is used by some firms and affects trading-day variation is the plan known as the 4-4-5 plan in which the first and second months of each quarter always contain exactly 4 weeks and the third month 5 weeks. This practice eliminates trading-day variation, since each period has exactly the same number of each type of day of the week as the corresponding periods in earlier years. When some activities are reported under such plans and others are reported on a strict calendar-month basis, the combined variation can be represented by the above example where the "B" weights represent the portion of the series that is independent of calendar composition.

From the information contained in the monthly data, we cannot actually determine how much activity occurs on each day of the week. All we can determine is how the monthly series varies with the composition of the calendar. For example, the activity represented by the Sunday "B" weight does not necessarily indicate that the activity occurs on Sunday. It may be distributed among the other days in some undeterminable fashion. Without recourse to other information, it is impossible to separate the various factors affecting the daily weights. What the weights do represent, after they are combined into monthly factors, is the monthly activity.

From the above discussion, it is apparent that the quality of a trading-day adjustment should be determined by its effect upon the monthly series, not by comparing the estimated weights with what is either supposed or known to be the actual daily rate of activity.

D. Constructing Monthly Trading-Day Adjustment Factors

The monthly trading-day adjustment factors are simply the estimated values, $\hat{I}_i$, shown in equation 3. These factors are divided into the data to remove the estimated trading-day variaton.

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[8] The difference between B1 and B2 is that activity represented by B1 varies with the length of the month. Since the seasonal removes length-of-month differences (except for differences between Februaries), variations explained by B1 and B2 are indistinguishable in the irregular component. (See footnote 10 for a further discussion.)

[9] The effect upon the Sunday weight may appear extreme since the Saturday is only 1/30 or 1/31 of the month. However, 4 full weeks, 28 days, are found in all months, and only 2 or 3 days are unique to each month. The shift of one day's activity is 1/2 or 1/3 of the monthly variation attributable to calendar composition.

Even though length-of-month variation can be removed by the seasonal, a common practice has been to include an allowance for it in the trading-day factors. When such factors are divided into the unadjusted data, followed by a recomputation of the seasonal factors, based upon the trading-day adjusted data, a compensating revision occurs in the seasonal factors. The same seasonally adjusted data, therefore, is obtained as would have been obtained if no allowance for the length of the month had been included in the trading-day factor. When it is desired to follow such a procedure, the following form is often used.[10]

(Equation 4)

$$\hat{I}_i' = \frac{X_{1i}b_1 + X_{2i}b_2 + \cdots + X_{7i}b_7}{30.4375},$$

where $30.4375 = \frac{365.25}{12}$ (the average length of month);

b_j's are those estimated in equation 2.

[10]Further consideration suggests that if part of the series is independent of the length of the month, equation 4 is incorrect conceptually for all months and equation 3 for Februaries.

In the example in section III, C, the "B" part of the series that does not vary with the calendar composition was explained in part by "B1" which varies only with the length of the month and "B2" which is constant from month to month. Statistically "B1" and "B2" can only be distinguished by comparing leap-year and non-leap-year Februaries, since the seasonal factor compensates for other differences in the length of month. This distinction was ignored in estimating the daily weights because only 1 out of every 48 values is involved.

To construct the adjustment factors when part of the series is independent of the length of the month, the following forms are correct:

Where a_1 is the portion of series dependent upon calendar composition and length of month (this corresponds to the "A" weights in section III, C);

a_2 is the portion of series dependent only upon the length of month, i.e., (B1);

a_3 is the portion of series which does not vary from month to month, i.e., (B2);

$a_1 + a_2 + a_3 = 1.0$;

N_i is either 31, 30, or 28.25;

c_j = the (A) part of the b_j weights in IIIc; and

$$a_1 = \frac{\sum_{1}^{7} c_j}{7};$$

then equation 4 becomes—

(Equation 4A)

$$\hat{I}_i'' = \frac{X_{1i}c_1 + X_{2i}c_2 + \cdots + X_{7i}c_7}{30.4375} + \frac{N_i a_2}{30.4375} + a_3,$$

Likewise, for leap-year Februaries, revise equation 3—

(Equation 3A)

$$\hat{I}_i''' = \frac{X_{1i}c_1 + X_{2i}c_2 + \cdots + X_{7i}c_7}{28.25} + \frac{29\,a_2}{28.25} + a_3;$$

and for non-leap-year Februaries,

(Equation 3A)

$$\hat{I}_i''' = \frac{X_{1i}c_1 + X_{2i}c_2 + \cdots + X_{7i}c_7}{28.25} + \frac{28\,a_2}{28.25} + a_3.$$

Since for a single series there are not enough observations to make a reliable determination of a_2 and a_3, it is necessary to make an arbitrary assumption. What evidence that has been obtained by comparing leap-year and non-leap-year Februaries suggests the variation is usually closer to $a_2 = 1.0 - a_1$ and $a_3 = 0.0$ than to $a_2 = 0.0$ and $a_3 = 1.0 - a_1$. The former is usually assumed and the equations reduce to those in the text. Such distinctions are academic in the seasonally adjusted series, since assuming either $a_2 = 0.0$ or $a_3 = 0.0$ affects only slightly the seasonally adjusted data for Februaries.

If one is interested in the trading-day adjusted data or in a study of the seasonal fluctuations, then the distinctions become important. For example, one could include the length-of-month variation in the seasonal pattern as did Kuznets (4). Or, if one wished to study seasonal patterns excluding length-of-month variation, it is necessary to attempt to estimate a_2 and a_3.

IV. RESULTS

A. Artificial Series

Tests with artificial series help us examine two questions: (1) What is the effect of the Census Bureau seasonal adjustment process upon known trading-day variation? and (2) How accurately can trading-day variation be estimated in the presence of various amounts of random variation?

To construct the artificial series, a set of monthly trading-day factors covering a period of 10 years were derived from daily weights in the manner described by equation 3. The following daily weights, which we shall refer to as "input weights," were used:

Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.	Total of weights
0.80	0.90	1.00	1.20	1.45	1.65	0.00	7.00

The set of monthly trading-day factors was taken as an artificial series. In addition, 18 artificial series were constructed by combining the monthly trading-day factors multiplicatively with 18 different random series designed to be normally distributed about a mean of 1.00. The average month-to-month change (without regard to sign) in the 18 random series ranged from 0.1 percent to 20.0 percent.

These artificial series resemble real economic series in two respects. First, they include a trading-day component that resembles the trading-day variation we might expect to find in some economic series. Second, the random components cover the range of variation most often found in the irregular components of economic series. In order to observe the effect of the seasonal adjustment process upon trading-day variation we did not introduce any seasonal or cyclical pattern, preferring to assume that they were constant. In this respect, the series do not resemble economic series and we may assume the test results to be somewhat better than if various seasonal and cyclical patterns had been introduced.

Three sets of daily weights were estimated from each artificial series. The first set was estimated directly from the artificial series and affords a basis for determining the effect of the seasonal adjustment process. Estimates corresponding with the first set could not be made from a real series which contained a seasonal component. The second set was estimated from the irregular components after seasonal adjustments of the artificial series were made with the X-9 version of Census Method II. If the seasonal adjustment process had no effect, the weights estimated before and after seasonal adjustment would be identical. The third set of weights was estimated from irregular components after iteration. Sometimes it has been considered necessary to iterate the estimation of the seasonal and trading-day variations in order to remove all the trading-day variation from the seasonal (see reference 5). If iteration improves the estimates, we would expect the third set of weights to correspond more closely to the input weights than do the second set. To obtain the third set,

Table 6.--DAILY WEIGHTS ESTIMATED FROM 10-YEAR ARTIFICIAL SERIES THAT CONTAIN KNOWN INPUT WEIGHTS AND A RANDOM COMPONENT

Series[1]	Average month-to-month percent change (without regard to sign) for--			Estimated daily weights for known input weights of--							Average deviation (without regard to sign) from input weights	Ratio of column 11			F-ratio[2]	Average standard error[2]
	Artificial series	Trading-day component	Random component	0.80 for Mon.	0.90 for Tues.	1.00 for Wed.	1.20 for Thurs.	1.45 for Fri.	1.65 for Sat.	0.00 for Sun.		b/a	c/b	c/a		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
1[3].......a				0.80	0.90	1.00	1.20	1.46	1.65	0.00	.002				326,000	.00
b	3.54	3.54	0.00	0.88	0.88	1.03	1.17	1.32	1.64	0.07	.054	27.00	.37	10.00	209	.05
c				0.84	0.90	1.01	1.17	1.41	1.65	0.02	.020				10,400	.01
2........a				0.81	0.89	1.01	1.20	1.46	1.64	-0.01	.01				12,600	.01
b	3.56	3.54	0.11	0.89	0.87	1.04	1.17	1.33	1.65	0.07	.05	5.00	.40	2.00	206	.05
c				0.85	0.89	1.01	1.18	1.41	1.65	0.00	.02				7,000	.01
3........a				0.90	0.82	1.08	1.11	1.55	1.58	-0.04	.08				160	.07
b	3.71	3.54	0.81	0.95	0.80	1.10	1.09	1.44	1.56	0.07	.09	1.12	.78	.88	140	.06
c				0.93	0.80	1.09	1.10	1.48	1.60	0.00	.07				420	.04
4........a				0.88	0.97	0.80	1.42	1.33	1.80	-0.22	.15				25.2	.18
b	4.61	3.54	2.74	0.84	0.96	0.81	1.39	1.28	1.76	-0.05	.12	.80	1.08	.87	28.4	.15
c				0.82	0.96	0.75	1.45	1.40	1.78	-0.14	.13				46.6	.14
5........a				1.10	0.64	1.34	1.03	1.49	1.74	-0.34	.22				20.3	.20
b	4.97	3.54	3.04	1.01	0.68	1.39	0.94	1.45	1.72	-0.19	.19	.86	1.26	1.09	27.9	.16
c				0.97	0.64	1.44	0.92	1.56	1.75	-0.29	.24				39.9	.15
6........a				0.75	0.60	1.36	0.59	1.95	1.11	0.64	.43				4.62	.33
b	6.09	3.54	5.30	0.95	0.45	1.44	0.55	1.79	1.19	0.62	.44	1.02	1.05	1.07	6.46	.25
c				0.89	0.46	1.46	0.51	1.91	1.18	0.59	.46				8.22	.24
7........a				0.70	1.62	0.19	2.28	0.73	1.39	0.09	.54				1.52	.70
b	9.91	3.54	9.66	1.21	1.10	0.56	1.99	0.68	1.51	-0.06	.40	.74	1.12	.83	2.36	.45
c				1.25	1.10	0.50	2.14	0.67	1.52	-0.18	.45				3.09	.45
8........a				3.41	0.43	0.30	0.89	1.68	1.17	-0.88	.81				.77	1.27
b	21.93	3.54	22.07	2.77	1.07	0.84	0.18	2.12	0.55	-0.52	.80	.99	1.11	1.10	.96	1.04
c				2.97	1.10	0.69	0.17	2.20	0.52	-0.65	.89				1.17	1.04

[1]Weights for "a" estimated from artificial series. Weights for "b" estimated from irregular component of X-9 seasonal adjustment of artificial series. Weights for "c" estimated from irregular component after iteration in which artificial series was adjusted by "b" weights prior to second X-9 seasonal adjustment.

[2]See section IV, C, for description of tests of significance and for the levels above which the F- and t-ratios indicate significant variation.

[3]Series contains only trading-day variation.

the artificial series were first adjusted for trading-day variation with the second set. Then the series were adjusted with X-9 to obtain improved estimates of the seasonals and trend-cycle upon which to base the irregular, from which the weights were reestimated.

The estimated weights and the average absolute deviations from the input weights are shown in table 6 for 8 of the 19 series. (Also shown are F-ratios and standard errors which are discussed in section IV, C.) From table 6 the following conclusions are made:

1. In general, the seasonal adjustment process does not have much effect. The weights estimated before and after seasonal adjustment are similar.

2. What effect the seasonal adjustment process does have is generally concentrated on the series with small random variations. For these series, the weights estimated after seasonal adjustment deviate more from the input weights than do those estimated before seasonal adjustment. For these series, it is possible to improve the weights estimated after seasonal adjustment by iteration.

3. There are indications that, for series with larger random variation, better estimates are made after seasonal adjustment than before. This may indicate that for highly irregular series the seasonal adjustment process dampens the irregular variations more than the trading-day variations and thereby improves the prospect for estimating the trading-day variation. For these series iteration appears to make the estimates slightly worse. To determine whether it actually makes them worse might require a test with more series.

These conclusions are similar to Census Bureau experience with economic series. Experience has also suggested that trading-day variation can be usefully estimated from the irregular component and that iteration often does not yield substantial improvements.[11]

Now let us consider our second question: How accurately can trading-day variation be estimated in the presence of random variation? It is apparent from examining the estimated daily weights and their average deviations from the input weights that when there are moderate or large random fluctuations, the estimates are quite poor. When $\bar{I}$ (the average month-to-month change, without regard to sign, in the irregular component) is above 5 percent, the deviation of the estimates from the input weights is greater than the variation among the input weights that we are trying to estimate.

[11]The improvement achieved by iteration also depends upon the sensitivity to trading-day variation of the moving averages used to estimate the seasonal component and whether the identification of extreme values is improved by iteration. These two factors are not considered in detail in this paper.

It is useful to extend this approach and determine theoretically the levels of $\bar{I}$ for various lengths of series, time periods, and trading-day patterns above which the deviation of the estimates from the input or true weights is greater than the variation among the input weights. Table 7 shows such levels of $\bar{I}$ above which—

(Equation 5)

$$E\left[\sum_{1}^{7} (b_j - B_j)^2\right] > \sum_{1}^{7} (B_j - 1.0)^2 ,$$

where b_j's are the estimated weights;

B_j's are the true or input weights (where $\sum_{1}^{7} B_j = 7$).

The critical level of $\bar{I}$ for our input weights with a 10-year series for the period 1953-63 is 6.9 percent and for a 6-year series for the period 1958-63, 5.5 percent. For the 5-day-week weight pattern and the same periods, it is 8.8 percent and 7.1 percent.

The levels of $\bar{I}$ shown in table 7 can be taken as theoretical upper limits above which trading-day variation cannot be usefully estimated. These limits actually are not of much practical use, since the true trading-day pattern is not known when applying the method to a real series. They do suggest, however, that estimates made from highly irregular series cannot be expected to be useful.

In making a decision about whether a particular series should be trading-day adjusted, the F-test discussed in section IV C is more useful than the theoretical relationship shown in table 7. The F-test does not require that the true trading-day pattern be known. Basing decisions on the F-test at the 5 percent or 1 percent confidence levels usually leads to the same decisions as those suggested by the levels of $\bar{I}$ in table 7. Sometimes the F-test tends to suggest significant variation at levels of $\bar{I}$ somewhat above those shown in table 7.

Regardless of the approach taken to the question of how accurately trading-day variation can be estimated, one conclusion emerges: The method cannot estimate trading-day variation in highly irregular series.[12]

[12]This conclusion also holds for other methods of making trading-day adjustments. Consider the possible gain in adjusting a highly irregular series for trading days. Assume that we are able to estimate the trading-day variation exactly and that we then adjust for it. For example, series 3 in table 6, a series with little irregular, has an average month-to-month change before seasonal or trading-day adjustment of 3.71 percent (column 1). An exact, or perfect trading-day adjustment would reduce the average change to 0.81 percent, a 78 percent reduction (column 3). For series 7, a highly irregular series, the adjustment would yield a much smaller reduction. The average change is reduced from 9.91 percent to 9.66 percent, only a 3 percent reduction, illustrating that even if the trading-day pattern could be estimated exactly, very little gain is possible. The error associated with any alternative method of trading-day adjustment is probably almost as large as the possible gain.

Table 7.—LEVELS OF IRREGULAR VARIATIONS ABOVE WHICH DEVIATION OF ESTIMATES FROM TRUE WEIGHTS IS GREATER THAN DEVIATION OF TRUE WEIGHTS FROM 1.0

Known daily weights								Average month-to-month percent change without regard to sign					
								Irregular component			Combined irregular and trading-day component		
Mon.	Tue.	Wed.	Thur.	Fri.	Sat.	Sun.	Variance	6-year series (1958-63)	10-year series (1953-62)	28-year series	6-year series (1958-63)	10-year series (1953-62)	28-year series
1.40	1.40	1.40	1.40	1.40	0.00	0.00	0.400	7.05	8.81	15.70	8.85	10.30	16.58
0.80	0.90	1.00	1.20	1.45	1.65	0.00	0.245	5.52	6.90	12.30	6.56	7.76	12.80
1.17	1.17	1.17	1.17	1.17	1.17	0.00	0.167	4.56	5.70	10.17	5.33	6.33	10.54
1.17	1.17	1.17	1.17	1.17	0.58	0.58	0.071	2.97	3.71	6.62	3.72	4.33	6.99

NOTE: See appendix B for the derivation of these measures.

B. Economic Series

Tests with economic series help us examine three related questions that cannot be readily answered with tests on artificial series: (1) How does the method compare with alternative techniques? (2) Do the estimates break down when applied to data outside the time period from which they were made? and (3) Is there evidence that the characteristics of trading-day variation change substantially over time or from one season to another and that our model which assumes no change, is inappropriate?

The criterion used to evaluate alternative trading-day adjustments of economic series is that the best method is the one resulting in the smallest month-to-month change, without regard to sign, in the irregular or unexplained variation (referred to as $\bar{I}$). It is not sufficient, however, to apply this criterion only for the "historical" period from which the estimates were made. A historical comparison is biased if the estimates of one or the other method explain, not only the trading-day variation, but part of the irregular variation. A more effective test is to apply the estimates made for the historical period to the current period, where methods which are too sensitive to the historical irregular fluctuations and those which inadequately allow for the characteristics of trading-day variation will both yield large fluctuations that are included in the computed $\bar{I}$.

Evaluation, therefore, consists of the following steps: (1) Estimate trading-day variation with each method from a historical period (For the regression method, estimates were made from irregular components of Method II, X-9 seasonal adjustments covering the historical period.) (2) make the trading-day adjustment to the historical and current data with the historical estimate; (3) obtain an irregular component by seasonally adjusting the combined historical and current data and compute $\bar{I}$ for each period (In each case the irregular component was from a Method II, X-9 seasonal adjustment of the combined historical and current data); and (4) compare the $\bar{I}$'s from the various methods giving particular attention to the current period.

Alternative trading-day adjustment methods are compared in table 8 for selected retail trade segments, bank debits, building permits and manufacturers' shipments and new orders for selected goods. A brief description of the results is as follows:

1. Retail sales.—For the period 1953-63, sales of eight retail kinds of business were adjusted for trading-day variation by regression estimates computed from the period 1953-61 and also 1957-61. These regression adjustments are compared with trading-day adjustments based upon average rates of sales on each day of the week computed from unpublished daily retail sales for 1962 that are available at the Bureau of the Census. They are also compared with series not adjusted for trading-days.

The regression estimates for 1953-61 yield the smallest $\bar{I}$'s, even for the current period of 1962-63 where we might expect the results to be biased in favor of the 1962 daily sales rates. The 1957-61 estimates are next best for both periods, followed by the daily sales rates and no adjustment.

Even though the unpublished estimates of the 1962 daily sales are not up to the Census Bureau publication standards,[13] the seven average daily rates are based on more evidence than is often available for an external adjustment and they appear to be reasonably close to what our experience would suggest as the daily sales pattern (see table 8). This comparison, therefore, supports the hypothesis that the customary external observation of the daily pattern of activity does not provide an adequate basis for a trading-day adjustment.

2. Bank debits.—For bank debits outside New York City, regression estimates computed from the period 1951-60 are compared with a 5-day week similar to the Federal Reserve adjustment used for part of this period. Also included are adjustments made with the Eisenpress method (3). By estimating a separate regression for each month, the Eisenpress method allows for seasonal characteristics in trading-day variation. It includes estimates for at least 12 coefficients (one for each month) and usually 24 (two for each month) or more, rather than seven. Two modifications of the original Eisenpress method are also compared. The first modification allows for a moving seasonal. The second, taking advantage of the evidence supplied by the Census Bureau regression, allows for the number of Mondays and Fridays as well as Saturdays and Sundays. The regression estimates yield smaller $\bar{I}$'s than the Eisenpress or 5-day week alternatives for both the current and historical period.

3. Building permits.—For "U.S. building permits," regression estimates computed from the period 1954-61 are compared with a 5-day week which might be selected a priori and with the series not adjusted for trading-days. The regression estimates and the 5-day-week adjustment yield approximately the same results for the historical period and for the current period of 1962-63. Both substantially reduce the irregular fluctuations found in the series not adjusted for trading days.

4. Manufacturers' shipments and new orders.—For manufacturers' shipments in four and new orders in five selected industries, regression estimates computed from the period 1953-61 are compared with two sets of weights that might be selected a priori: (1) A 5-day week and (2) weights where Saturday and Sunday receive partial weights. They are also compared with the series not adjusted for trading days. The a priori weights where Saturday and Sunday receive partial weights have been found to be appropriate for the aggregate series and thus might be expected to be appropriate for each component.

The regression estimates yield the smallest $\bar{I}$ in the historical period for seven of the nine series. In the current period, 1962-63, they are best for three series while the a priori weights where Saturday and Sunday receive partial weights are best for three series and the a priori 5-day week weights are best for three series.

Some of these manufacturing series contain larger irregular variations than do most of the other test series. Where the irregular variations are large, it is possible to discern the tendency for the differences between alternatives to be small relative to the magnitude of $\bar{I}$. The results are less conclusive, illustrating the fact that as irregular variations increase, the possible gains from trading-day adjustments decrease.

On the basis of these tests our conclusion is that the regression method performs quite well in comparison with other methods. There is little evidence that the estimates break down substantially in the current period or that changes in the characteristics of the trading-day variation over time or from one season to another invalidate our stable model.

C. Tests of Significance

The regression routine being added to Census Method II includes two tests which are useful guides in determining

[13] The 1962 daily sales were voluntarily reported by about a fourth of the 1,600 respondents in the weekly survey of stores with 10 or less outlets. The number of reports by kind of business is quite small, ranging from 2 for liquor stores and 8 for variety stores to 36 for lumber and building material dealers and 38 for eating and drinking places. The daily data, therefore, contains some inaccuracies. From the daily sales figures seven daily weights were computed by determining, for each week, the percentage of the week's sales occurring on each day and then averaging the daily percentages over the 52 weeks to arrive at the daily rates.

Table 8.--COMPARISON OF ALTERNATIVE TRADING-DAY ADJUSTMENTS FOR SELECTED BUSINESS ACTIVITIES

Series	Daily weights								Ī, average monthly change, without regard to sign, in irregular component				Rank (lowest Ī equals one)		F-ratio[1]	Average standard error
	Total	Mon.	Tue.	Wed.	Thur.	Fri.	Sat.	Sun.					Historical	Current		
									1953-61	1961	1962	1963	1953-61	1962-63		
RETAIL SALES																
Eating and drinking places:																
Regression, 1953-61..........	7.00	0.95	1.04	0.97	0.91	1.12	1.10	0.92	.76	.52	.72	.78	2	2	2.25	.04
Regression, 1957-61..........	7.00	0.86	1.02	0.98	0.92	1.15	1.07	1.00	.74	.55	.53	.78	1	1	1.60	.04
1962 daily sales.............	7.00	0.92	0.92	0.97	1.02	1.36	1.32	0.49	1.05	.94	.84	.99	4	4	14.42	x
No adjustment[2]...............	-	-	-	-	-	-	-	-	.88	.79	.75	.95	3	3	9.23	x
Variety stores:																
Regression, 1953-61..........	7.00	0.96	0.96	0.97	1.01	1.15	1.59	0.35	1.54	.76	.83	.72	1	1	1.24	.10
Regression, 1957-61..........	7.00	0.90	1.11	0.94	0.85	1.30	1.56	0.34	1.55	.99	.83	.82	2	2	1.07	.12
1962 daily sales.............	7.00	1.04	0.94	0.87	0.94	1.32	1.88	0.01	1.61	1.38	1.18	.88	3	3	2.14	x
No adjustment[2]...............	-	-	-	-	-	-	-	-	2.61	1.96	1.75	1.58	4	4	18.07	x
Men's and boys' wear stores:																
Regression, 1953-61..........	7.00	0.79	1.02	1.13	0.70	1.59	1.43	0.34	2.27	2.65	2.82	2.70	1	2	.41	.14
Regression, 1957-61..........	7.00	0.93	0.75	1.18	0.97	1.45	1.09	0.62	2.33	1.87	3.04	3.36	2	4	2.11	.18
1962 daily sales.............	7.00	1.04	0.97	0.94	1.15	1.36	1.53	0.01	2.46	2.87	2.21	2.39	3	1	.99	x
No adustment[2]...............	-	-	-	-	-	-	-	-	2.96	3.38	3.02	2.80	4	3	15.39	x
Shoe stores:																
Regression, 1953-61..........	7.00	0.78	1.07	0.91	0.74	1.57	1.82	0.11	2.23	2.33	2.86	3.05	1	3	.63	.14
Regression, 1957-61..........	7.00	0.42	1.30	1.15	0.48	1.86	1.27	0.52	2.44	2.08	2.77	3.08	2	2	2.82	.21
1962 daily sales.............	7.00	0.95	0.87	0.76	0.91	1.28	2.19	0.04	2.45	2.26	2.99	3.35	3	4	4.28	x
No adjustment[2]...............	-	-	-	-	-	-	-	-	3.32	1.52	1.39	3.20	4	1	26.91	x
Furniture, home furnishings stores:																
Regression, 1953-61..........	7.00	1.24	0.92	0.84	1.23	1.10	1.11	0.57	1.25	1.14	1.08	2.15	1	1	.48	.07
Regression, 1957-61..........	7.00	1.23	0.89	0.97	1.14	1.15	1.07	0.55	1.27	1.08	1.26	2.06	2	3	1.30	.09
1962 daily sales.............	7.00	1.30	1.04	0.95	1.13	1.18	1.39	0.01	1.45	1.28	1.88	1.36	3	2	6.74	x
No adjustment[2]...............	-	-	-	-	-	-	-	-	1.75	1.36	1.46	2.26	4	4	12.95	x
Lumber yards, and building materials dealers:																
Regression, 1953-61..........	7.00	1.13	1.11	1.16	1.22	1.15	0.67	0.56	1.69	1.32	.86	.88	1	1	.29	.10
Regression, 1957-61..........	7.00	1.00	0.99	1.40	1.11	1.24	0.64	0.63	1.77	1.28	1.01	1.09	2	2	1.15	.14
1962 daily sales.............	7.00	1.29	1.26	1.16	1.22	1.19	0.87	0.01	1.81	1.74	1.76	1.10	3	3	3.56	x
No adjustment[2]...............	-	-	-	-	-	-	-	-	2.86	2.94	2.15	2.46	4	4	19.79	x
Hardware stores:																
Regression, 1953-61..........	7.00	1.08	0.96	0.78	1.29	1.22	1.15	0.51	1.58	1.81	1.80	2.03	1	1	1.02	.10
Regression, 1957-61..........	7.00	0.96	0.84	0.96	1.19	1.44	0.90	0.71	1.62	1.73	2.24	2.20	2	3	2.14	.15
1962 daily sales.............	7.00	1.11	1.04	1.10	1.05	1.22	1.48	0.00	2.22	2.47	2.24	2.12	3	2	5.02	x
No adjustment[2]...............	-	-	-	-	-	-	-	-	2.79	2.83	2.25	3.09	4	4	15.02	x
Liquor stores:																
Regression, 1953-61..........	7.00	0.87	0.96	0.75	1.04	1.32	1.53	0.52	1.15	.70	1.05	1.06	2	2	1.36	.07
Regression, 1957-61..........	7.00	0.78	1.05	0.79	0.90	1.46	1.53	0.50	1.11	.61	.88	.71	1	1	1.06	.08
1962 daily sales.............	7.00	1.24	0.76	0.83	0.86	1.15	2.17	0.00	1.55	1.56	1.78	1.39	3	3	13.97	x
No adustment[2]...............	-	-	-	-	-	-	-	-	2.46	1.80	2.08	2.52	4	4	57.34	x
Average rank of retail sales; 8 components:																
Regression, 1953-61..........	x	x	x	x	x	x	x	x	x	x	x	x	1.2	1.6	x	x
Regression, 1957-61..........	x	x	x	x	x	x	x	x	x	x	x	x	1.8	2.2	x	x
1962 daily sales.............	x	x	x	x	x	x	x	x	x	x	x	x	3.1	2.8	x	x
No adjustment	x	x	x	x	x	x	x	x	x	x	x	x	3.9	3.4	x	x

Table 8.--COMPARISON OF ALTERNATIVE TRADING DAY ADJUSTMENTS FOR SELECTED BUSINESS ACTIVITIES--Con.

Series	Daily weights: Total	Mon.	Tue.	Wed.	Thur.	Fri.	Sat.	Sun.	Ī, average monthly change, without regard to sign, in irregular component				Rank (lowest Ī equals one): Historical	Current	F-ratio[1]	Average standard error
									1951-60	1961	1962	1963	1951-60	1961-63		
BANK DEBITS																
Regression, 1951-60	7.00	1.52	1.14	1.24	1.24	1.38	0.19	0.29	.90	.56	.48	[3].62	1	1	1.56	.07
Regression, 1951-60[4]	7.00	1.53	1.06	1.31	1.19	1.46	0.14	0.32	.88	.60	.53	[3].72			2.16	.20
Eisenpress, 1951-60[5]	-	-	-	-	-	-	-	-	1.30	1.74	1.86	[3]2.54	4	5	3.77	x
Eisenpress[5] [6]	-	-	-	-	-	-	-	-	.95	1.02	.61	[3]1.62	3	4	5.95	x
Eisenpress[5] [7]	-	-	-	-	-	-	-	-	.90	1.12	.40	[3]1.12	2	3	2.77	x
A priori weights	7.00	1.40	1.40	1.40	1.40	1.40	0.00	0.00	1.39	.88	.72	[3].78	5	2	21.25	x
No adjustment[2]	-	-	-	-	-	-	-	-	3.71	3.39	3.02	[3]4.88	6	6	143.74	x
									1954-61		1962	1963	1954-61	1962-63		
BUILDING PERMITS																
Regression, 1954-61	7.00	1.20	1.11	1.48	1.25	1.60	0.32	0.69	3.17		2.55	3.28	2	1	.91	.17
A priori	7.00	1.40	1.40	1.40	1.40	1.40	0.00	0.00	3.13		3.81	3.62	1	2	2.42	x
No adjustment[2]	-	-	-	-	-	-	-	-	5.19		3.38	7.08	3	3	33.90	x
									1953-61		1962	1963	1953-61	1962-63		
MANUFACTURERS' SHIPMENTS AND NEW ORDERS																
Other petroleum products shipments:																
Regression, 1953-61	7.00	1.13	1.19	1.01	1.24	1.11	0.84	0.46	1.39		1.61	1.87	1.5	2	.36	.08
A priori (1)	7.00	1.40	1.40	1.40	1.40	1.40	0.00	0.00	3.29		2.58	4.46	4	4	59.59	x
A priori (2)	7.00	1.17	1.17	1.17	1.17	1.17	0.58	0.58	1.39		1.89	1.32	1.5	1	2.60	x
No adjustment[2]	-	-	-	-	-	-	-	-	2.42		3.18	2.81	3	3	29.89	x
Construction, mining, and material handling machinery shipments:																
Regression, 1953-61	7.00	0.95	1.53	0.93	0.95	1.46	0.64	0.54	2.63		2.00	1.17	1	2	.15	.16
A priori (1)	7.00	1.40	1.40	1.40	1.40	1.40	0.00	0.00	3.95		2.66	4.32	4	4	13.80	x
A priori (2)	7.00	1.17	1.17	1.17	1.17	1.17	0.58	0.58	2.86		1.84	1.21	2	1	1.72	x
No adjustment[2]	-	-	-	-	-	-	-	-	3.65		2.92	2.10	3	3	9.11	x
Tobacco shipments:																
Regression, 1953-61	7.00	1.89	0.96	1.42	1.18	1.24	0.00	0.31	1.54		1.28	1.89	1	1	2.63	.11
A priori (1)	7.00	1.40	1.40	1.40	1.40	1.40	0.00	0.00	2.68		2.40	2.22	2	2	21.80	x
A priori (2)	7.00	1.17	1.17	1.17	1.17	1.17	0.58	0.58	2.92		2.43	3.83	3	3	32.90	x
No adjustment[2]	-	-	-	-	-	-	-	-	4.35		3.68	6.23	4	4	76.05	x
Complete aircraft shipments:																
Regression, 1953-61	7.00	0.81	1.08	0.94	1.66	1.34	0.56	0.60	3.66		6.18	5.82	1	3	.80	.18
A priori (1)	7.00	1.40	1.40	1.40	1.40	1.40	0.00	0.00	4.71		5.16	6.15	4	1	10.00	x
A priori (2)	7.00	1.17	1.17	1.17	1.17	1.17	0.58	0.58	3.98		5.10	6.49	2	2	3.46	x
No adjustment[2]	-	-	-	-	-	-	-	-	4.69		6.07	6.48	3	4	14.77	x
Average rank of manufacturers shipments; 4 components, Ī >5.00:																
Regression, 1953-61	x	x	x	x	x	x	x	x	x		x	x	1.1	2.0	x	x
A priori (1)	x	x	x	x	x	x	x	x	x		x	x	3.5	2.8	x	x
A priori (2)	x	x	x	x	x	x	x	x	x		x	x	2.1	1.8	x	x
No adjustment	x	x	x	x	x	x	x	x	x		x	x	3.3	3.5	x	x

Metal working machinery, new orders:															
Regression, 1953-61..........	7.00	1.33	0.39	1.93	0.91	1.62	1.04	-0.21	9.29	10.99	13.58	1	2	.12	.49
A priori (1)................	7.00	1.40	1.40	1.40	1.40	1.40	0.00	0.00	9.32	11.65	12.75	2	1	3.59	x
A priori (2)................	7.00	1.17	1.17	1.17	1.17	1.17	0.58	0.58	9.42	10.24	16.08	3	3	2.03	x
No adjustment[2]..............	-	-	-	-	-	-	-	-	9.93	10.04	17.60	4	4	3.54	x
Motor vehicle parts, new orders:															
Regression, 1953-61..........	7.00	1.04	1.12	1.83	0.45	2.44	0.37	-0.26	10.84	5.84	4.88	2	1	.21	.48
A priori (1)................	7.00	1.40	1.40	1.40	1.40	1.40	0.00	0.00	11.07	6.67	5.85	4	3	3.46	x
A priori (2)................	7.00	1.17	1.17	1.17	1.17	1.17	0.58	0.58	10.87	6.38	5.76	3	2	2.57	x
No adjustment[2]..............	-	-	-	-	-	-	-	-	10.80	6.97	6.68	1	4	5.90	x
Blast furnaces, steel mills, new orders:															
Regression, 1953-61..........	7.00	0.52	1.62	1.21	1.44	1.13	0.72	0.35	10.27	10.43	12.69	1	2	.32	.43
A priori (1)................	7.00	1.40	1.40	1.40	1.40	1.40	0.00	0.00	10.79	12.66	9.37	3	1	2.31	x
A priori (2)................	7.00	1.17	1.17	1.17	1.17	1.17	0.58	0.58	10.72	10.90	12.31	2	3	.86	x
No adjustment[2]..............	-	-	-	-	-	-	-	-	11.54	11.63	12.19	4	4	4.06	x
Communication equipment, new orders:															
Regression, 1953-61..........	7.00	1.17	1.72	-0.46	2.10	1.43	1.40	-0.37	11.05	12.92	5.50	1	1	.24	.56
A priori (1)................	7.00	1.40	1.40	1.40	1.40	1.40	0.00	0.00	12.86	12.37	6.19	3	2	4.78	x
A priori (2)................	7.00	1.17	1.17	1.17	1.17	1.17	0.58	0.58	12.69	13.55	6.34	2	3	4.14	x
No adjustment[2]..............	-	-	-	-	-	-	-	-	13.00	15.23	7.41	4	4	3.53	x
Electrical industrial apparatus new orders:															
Regression, 1953-61..........	7.00	1.02	1.71	-0.87	3.27	0.84	0.40	0.63	9.54	5.97	5.02	1	2	1.28	.52
A priori (1)................	7.00	1.40	1.40	1.40	1.40	1.40	0.00	0.00	10.95	6.08	5.16	2	3	4.11	x
A priori (2)................	7.00	1.17	1.17	1.17	1.17	1.17	0.58	0.58	11.04	6.22	4.38	3	1	4.49	x
No adjustment[2]..............	-	-	-	-	-	-	-	-	11.42	7.54	4.65	4	4	5.17	x
Average rank of manufacturers' new orders; 5 components, $\bar{I} > 10.00$:															
Regression, 1953-61..........	x	x	x	x	x	x	x	x	x	x	x	1.2	1.6	x	x
A priori (1)................	x	x	x	x	x	x	x	x	x	x	x	2.8	2.0	x	x
A priori (2)................	x	x	x	x	x	x	x	x	x	x	x	2.6	2.4	x	x
No adjustment	x	x	x	x	x	x	x	x	x	x	x	3.4	4.0	x	x

x--Not applicable.

[1]The F-ratio shown for "No adjustment" was computed from the combined irregular-trading day component. It is a test for the presence of trading-day variation in the unadjusted data. F-ratios shown for the other adjustments were computed from irregular components after the series was adjusted with the daily weights. Thus, they are a test for residual trading-day variation, that is, for the adequacy of the daily weights. Section IV, C describes these tests of significance and shows levels above which these ratios indicate significant variation.

[2]"No adjustment" is shown without weights. An alternative would be to show a weight of 1.0 for each day of the week.

[3]Excludes the extreme value of November 1963. Although using the October to November and November to December changes results in higher $\bar{I}$'s for 1963, it does not affect the current rankings.

[4]No extremes deleted from regression in order to be comparable with Eisenpress regressions.

[5]The Eisenpress method does not yield 7 daily weights. A separate regression is made for each month.

[6]Modified to include a moving seasonal.

[7]Modified to include a moving seasonal and dummy variables of Saturday-Sunday combined and Monday-Friday combined where X_1 = 0, 1, or 2 depending on whether there are 8, 9, or 10 Saturdays and Sundays in a month and X_2 = 0 or 1 depending on whether there are 8 or 9 Mondays and Fridays in a month. This is in contrast to the variables used by Eisenpress where X_1 = 0 or 1 depending on whether there are 4 or 5 Saturdays a month and X_2 = 0 or 1 depending on whether there are 4 or 5 Sundays in a month.

whether trading-day variation is present in a series. One of these is a standard F-test to determine the significance of the regression and the other a t-test to determine if a daily weight differs significantly from a specified value. These tests are described more fully in appendix A.

The F-ratio and the average of the standard errors for the daily weights[14] are shown in tables 6 and 8 to provide a convenient reference when assessing results of other series. Under assumptions of normality, the F-ratio and t-ratio indicate significant variation when they are above the levels shown in table 9.

The assumptions of such tests are not always exactly fulfilled. In particular, the seasonal-adjustment process results in an irregular component which tends to be autocorrelated, even if the original series was an artificial series with a random irregular. Also, and perhaps more importantly, the adjustment process, by identifying values outside a specified sigma limit as extreme and excluding them from the computation of the daily weights, sometimes excludes some good values or does not succeed in excluding all extremes. Including or excluding a few values in the tail of the distribution can substantially affect the residual variance and the F- and t-ratios. For example, in table 8, values outside 2 sigma were designated as extreme for the retail sales series. If the limit had been 2.8 sigma instead, the F-ratios for "Eating and drinking places" would have been 1.63, .76, 9.45 and 8.34 rather than 2.25, 1.60, 14.42 and 9.23. In table 6, no extremes were deleted from the artificial series since the random values all belong to the same distribution. If a 2.8 or 2.0 sigma limit had been used, the F-ratio for series 6a would have been 6.02 or 8.05 rather than 4.62; for series 7a the F-ratio would have been 1.66 or 2.38 rather than 1.52.

In general, a choice of a sigma limit of about 2.8 appears reasonable in the new Method-II routine. In some instances, however, a limit of 2.0 or even less is required to identify the extremes. The analyst, therefore, should give close attention to the choice of the limit and to whether the assumption of a normal distribution provides valid test results.

In spite of such violations of the assumptions upon which the tests are based, the tests are of assistance in developing a good trading-day adjustment.

V. LIMITATIONS

A. Reliability

An important limitation of the regression and other methods is the amount of error associated with the estimates. This is discussed in section IV and will not be considered further.

B. Variation Due to Holidays

Retail sales increase in the fall because of the occurrence of Christmas and, at the manufacturing level, dips occur in many series during July, because of the Fourth of July and plant-wide vacations. Most such variations due to holidays are accounted for by the seasonal factors but, in some instances, the effect of a holiday is not the same each year, varying with the calendar composition. In these cases, a residual variation, referred to simply as holiday variation, is present in the irregular component which, if it is not recognized and allowed for, can distort the trading-day estimates. Its estimation and removal can also reduce the irregular variation. Our method is limited in that it does not include an allowance for holiday variation. A sequential adjustment, however, can be made for holidays.

Now that a trading-day adjustment method has been computerized, it may be possible to study more thoroughly the relation of trading-day and holiday variation and develop techniques that allow for both variations simultaneously. Following is a summary of our experience to date:

1. Holiday variation is unimportant in many series that contain trading-day variation and also in most or all series that do not contain trading-day variation.[15] Where there is apparent holiday variation, it does not seem to seriously distort the estimates of trading-day variation, with the possible exception of Easter in series such as the retail sales of apparel.

2. The practice of assigning a zero weight to holidays when constructing trading-day adjustment factors is incorrect. The essential element is not whether the activity is shut down for the holiday, but whether the variation in the monthly series is related to the holiday.[16]

3. With the exception of Easter, major U.S. holidays are correlated with calendar composition in such a way that it is difficult to separate trading-day and holiday variation. For example, every time Christmas falls on Monday, December begins on Friday and contains five Fridays, Saturdays, and Sundays.

4. The effect of a holiday often occurs in 2 adjacent months. For example, when Labor Day is early, some "back to school" shopping occurs in August, but when Labor Day is late, such shopping occurs in September.

[14]Since the standard errors of the 7 daily weights are approximately equal in practice, only their average is shown.

[15]One type of series where holiday variation can be a problem is that based upon a survey covering one week of the month. For example, the series on the average workweek can be affected by holidays that fall in the survey week.

[16]The Federal Reserve Board discontinued this practice for the Index of Industrial Production in 1953. "It is not always clear that holidays have an impact on output proportional to their number in the month, as was assumed under the old procedure. In some cases output 'lost' on account of holidays may be made up on contiguous days, particularly where the rate of purchase or consumption of the product is not influenced by the holiday. In other cases, as in connection with Christmas Day or July 4, output losses may be more than proportional to the 1 day of holiday time" (2, p. 1261). The current Federal Reserve practice is to make no allowance for holidays.

Table 9.--SIGNIFICANT F- AND t-RATIOS FOR SERIES OF VARIOUS LENGTHS

Length of series	Total degrees of freedom	Regression degrees of freedom	Residual degrees of freedom	F		t (2-tailed test)	
				5 Percent	1 Percent	5 Percent	1 Percent
5 years	60	6	54	2.27	3.15	2.00	2.66
6 years	72	6	66	2.24	3.09	2.00	2.66
7 years	84	6	78	2.21	3.04	1.98	2.62
8 years	96	6	90	2.19	2.99	1.98	2.62
10 years	120	6	114	2.17	2.95	1.98	2.62
12 years	144	6	138	2.16	2.92	1.96	2.58
16 years or more (n years)	12n	6	12n-6	2.14	2.90	1.96	2.58

5. The only series seasonally adjusted by Census where significant holiday variation has been found are sales of certain types of retail business where adjustments are currently being made for Easter, Labor Day, and Thanksgiving-Christmas. For Easter a technique similar to the standard Easter adjustment is used, see references 5 and 8. A similar technique is used for Labor Day and Thanksgiving-Christmas. Essentially, it consists of arranging the irregular component, after adjustment for trading days, in an order that appears to bear a relationship to the date of the holidays and then fitting a smooth curve to estimate the relationship. In the case of Easter, the relationship between the date of the holiday and the variation in the data is obvious, the later Easter occurs, the higher are April sales and the lower are March sales and vice versa. In other instances, the relationship, if it exists, is not so obvious either a priori or in the data.

C. Changes Over Time

Our method of estimating trading-day variation makes no provision for changes in the characteristics of trading-day variation over a period of years. It is based upon the assumption that trading-day variation is a fairly constant, deep-seated phenomenon in the economy. However, there are several factors in the economy which can be assumed to cause changes in trading-day variation: (a) A half day of work or no work on Saturday is more common now than a few years ago; (b) fewer banks are open on Saturdays than before; (c) more retail stores are open evenings than in the past; (d) the amount of overtime worked on Saturdays, Sundays, and other days varies over the business cycle; (e) the introduction of electronic computers has changed bookkeeping practices in various industries and collection and processing of data in organizations such as the Bureau of the Census.

A relatively simple way of handling possible changes over time in trading-day variation is to restrict the analysis to a reasonably short period. At the Census Bureau, a period of about 8 or 10 years is usually used. (For the retail sales adjustments shown in table 8, better results were obtained for the years 1962-63 when estimates were made from 1953-61 rather than restricting the period to 1957-61. In this case, the gain from including 4 years, 1953-56 is larger than any loss arising from changes in the trading-day variation between 1953-56 and 1957-61.) It appears doubtful that including an explicit allowance for changes over time is necessary.[17]

In new Census Bureau seasonal-adjustment programs various options will be available to control the time period upon which the computations are based. For example, if the series covers 1948-64, the trading-day regression could be computed from say, 1957-64 and if the results are significantly different than previous results computed from say, 1953-60, the new estimates would be applied. Such options should be an adequate tool for allowing for changes over time when necessary.

[17]Canadian retail sales, however, shows some evidence of changes over time in trading-day variation, reported in reference 5.

D. Changes Within the Year

Our method of estimating trading-day variation assumes that the characteristics of trading-day variation are the same in each season of the year. In some instances, this assumption may not be entirely warranted. For example, many department stores have different hours of business in summer months than in the other months of the year.

Where there is a seasonal pattern in the trading-day variation it is possible to adapt the above method by separating some months from others and developing two or more sets of daily weights. Such procedures, though, are pretty much restricted by the lack of sufficient data to provide reliable estimates.

REFERENCES

1. Bongard, J., "Notes on the Characteristics of Seasonal Variation In Europe," Seasonal Adjustment on Electronic Computers, Organization for Economic Cooperation and Development, 1960, pp. 179-192.
2. Broida, Arthur L., et al, "Revised Federal Reserve Monthly Index of Industrial Production," Federal Reserve Bulletin, vol. 39, no. 12, December 1953, pp. 1260-1264.
3. Eisenpress, Harry, "Regression Techniques Applied to Seasonal Corrections and Adjustments for Calendar Shifts," Journal of the American Statistical Association, vol. 51, no. 276, December 1956, pp. 615-620.
4. Kuznets, Simon, Seasonal Variations in Industry and Trade, National Bureau of Economic Research, Inc., 1933, pp. 10, 25, New York.
5. Marris, Stephen N., "The Measurement of Calendar Variation," Seasonal Adjustment on Electronic Computers, Organization for Economic Cooperation and Development, 1960, pp. 345-360.
6. Method of Adjusting Time Series for Inequalities in the Number and Nature of Working Days, Research Paper No. 6, Organization for Economic Cooperation and Development, ESD/S/228, February 1961.
7. Shiskin, Julius, Electronic Computers and Business Indicators, Occasional Paper 57, National Bureau of Economic Research, Inc., 1957.
8. Thompson, Orville K., and Smith, Howard G., "Revised Indexes of Department Store Sales and Stocks," Federal Reserve Bulletin, vol. 37, no. 12, December 1951, pp. 1463-1515.
9. U.S. Bureau of the Census, Specifications for the X-9 version of Census Method II seasonal-adjustment program. Office of the Chief Economic Statistician, Bureau of the Census, Washington, D.C., 20233. (Specifications for the X-11 version that will include the trading-day routine will be made available.)
10. Young, Allan, "Census Trading-Day Adjustment Method," Business Cycle Developments, U.S. Bureau of the Census, May 1964, pp. 59-64. (The present paper covers the same ground and is a more detailed discussion.)

Appendix A. Mathematical Statement of Regression Method

The regression method of estimating daily weights rests on two fundamental assumptions:

(Assumption 1) All residual trading-day variation appears in the irregular; and

(Assumption 2) This, variation may be expressed in terms of 7 daily weights.

The steps in the method are as follows:

Step 1.—Make a preliminary seasonal adjustment to obtain an irregular component, $\bar{I}$, which is assumed to contain all trading-day variation.

Step 2.—Delete extreme values in $\bar{I}$ which would tend to distort the estimated daily weights.[1] Call the resulting modified irregular I^*. In Census Method II, the mean of I^* is approximately 100.0.

Step 3.—Divide each I^* by 100, multiply by the number of days in that month, and subtract the number of days in the month so that the estimated daily weights will sum to zero.[2] Call this transformed series Y.

We assume that—

(Assumption 3) $Y = XB + E$,

where $Y = [Y_1\ Y_2 \ldots Y_n]'$ is the vector of the transformed irregular component and trading-day variation and n is the number of months included in the regression,

$E = [E_1\ E_2 \ldots E_n]'$ is the vector of the "true" irregular series,

$B = [B_1\ B_2 \ldots B_7]'$ is the vector of the daily weights to be estimated[3] and $\sum_1^7 B_j = 7$,

X is the matrix of independent variables with X_{1i}, X_{2i}, ..., X_{7i} corresponding to the number of Mondays, Tuesdays, ..., Sundays in a given month.

Step 4.—Modify the weights to sum to 0 for purposes of testing for the existence of trading-day variation. Since $\sum_1^7 B_j = 0$ by definition, $B_7 = 0 - \sum_1^6 B_j$. Define $B_1, \ldots, B_6$ as the Monday, ..., Saturday weights and B_7 as the Sunday weight. Hence, $Y = X B + E$ becomes $Y = \hat{X}\hat{B} + E$,

where $\hat{B} = [B_1\ B_2 \ldots, B_6]'$

and $\hat{X}_{ji} = X_{ji} - X_{7i}$ $(j = 1, \ldots, 6;\ i = 1, \ldots, n)$.

Then

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ \cdot \\ Y_n \end{bmatrix} = \begin{bmatrix} \hat{X}_{11} & \hat{X}_{21} & \cdots & \hat{X}_{61} \\ \hat{X}_{12} & \hat{X}_{22} & \cdots & \hat{X}_{62} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \hat{X}_{1n} & \hat{X}_{2n} & \cdots & \hat{X}_{6n} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \cdot \\ \cdot \\ \cdot \\ B_6 \end{bmatrix} + \begin{bmatrix} E_1 \\ E_2 \\ \cdot \\ \cdot \\ \cdot \\ E_n \end{bmatrix}$$

or

$$Y_i = \sum_1^6 \hat{X}_{ji} B_j + E_i \qquad (i = 1, 2, \ldots n).$$

Step 5.—Assume:

(Assumption 4) $E(E) = O$, where O is an n-term vector of zeroes;

(Assumption 5) $V(E) = \sigma^2 I$, where I is an n x n identity matrix and σ^2 is the variance of the E_i;

(Assumption 6) $\hat{X}$ is fixed;

(Assumption 7) $\hat{X}$ has rank = 6.

Compute $\hat{b} = [b_1\ b_2 \ldots b_6]' = (\hat{X}'\hat{X})^{-1}\hat{X}'Y$, which is the least-squares estimate of $\hat{B}$. As an estimate of B_7, form $b_7 = 0 - \sum_1^6 b_j$.

Step 6.—Estimate the standard error of each weight as follows:

$$S_{b_j} = \left[\frac{(\hat{X}'\hat{X})^{-1}_{jj} \cdot e'e}{n-6} \right]^{\frac{1}{2}} \qquad (j = 1, 2, \ldots, 6),$$

where $e = [e_1\ e_2 \ldots, e_n]' = Y - \hat{X}\hat{b}$ is the vector of residuals from the fitted regression.

Now

$\sigma^2_{b_7} = \sum_1^6 \sigma^2_{b_j} + 2 \sum_1^6 \sum_{\substack{1 \\ j \neq j'}}^6 \sigma_{b_j b_{j'}}$. As an estimate of $\sigma^2_{b_7}$, form $S^2_{b_7} = \frac{e'e}{n-6} \sum_1^6 \sum_1^6 (\hat{X}'\hat{X})^{-1}_{ji}$.

Step 7.—To make inferences about the estimated weights b_j, assume that—

(Assumption 8) The E_i have a joint normal distribution.

To test whether b_j differs from 0, form $t_{b_j} = b_j/S_{b_j}$, which has a t-distribution with n-6 degrees of freedom. To test whether b_j differs from any specified k, form

$t'_{b_j} = (b_j - k)/S_{b_j}$, which has the same distribution as t_{b_j}.

A test for the existence of trading-day variation in I^* may be made as follows. Form

$$F = \frac{\hat{b}'\hat{X}'\hat{X}\hat{b}/6}{e'e/(n-6)},$$

which has an F-distribution with 6 and n-6 degrees of freedom. If this ratio is sufficiently low to conclude that the regression is not significant, we may conclude that there is no trading-day variation present.

To derive weights to be used in the seasonal program, add 1 to each b_j. If a trading-day adjustment is made prior to the seasonal adjustment, the b_j explain the residual trading-day variation. To combine the b_j with prior weights, use the formula $D_j = P_j + b_j$ where D_j are the final weights and P_j are the prior weights. Use the standard errors of the b_j as the standard errors of the D_j for inferences about the D_j.

[1] The specific procedure for identifying extreme values will be included in specifications for new Census Bureau seasonal programs. Essentially it consists of removing values where the residual computed in the regression falls outside a given sigma limit and then recomputing the regression. In section IV, C of the text it is suggested that sigma limits of 2.0 or 2.8 are often satisfactory.

[2] Februaries are handled in a special manner. The seasonal adjustment procedure eliminates variation due to the length of the month by, in effect, dividing each monthly value other than Februaries by 30 or 31. To reintroduce this length-of-month weighting, simply multiply each I^* by 30 or 31. Februaries are divided in the seasonal program by the average length of February for the period covered, which will be approximately 28.25. Hence, Februaries are multiplied here by 28.25. However, the actual number of days in the month (28 or 29) is subtracted from the result.

[3] The constant term in the regression is forced to equal zero.

Tests on artificial series suggest that assumptions 1 and 4-7 are not seriously violated. Assumption 2 may be violated when a single set of daily weights is estimated from a series where the trading-day pattern changes seasonally, cyclically or secularly.

Assumption 3 states that the residual trading-day and "true irregular" parts of the transformed irregular Y are related in an additive fashion ($Y = XB + E$). However, the daily weights estimated from the regression are combined into monthly calendar adjustment factors, which are used to remove trading-day variation in the unadjusted series in a multiplicative fashion. While it may seem inconsistent to estimate trading-day variation additively and apply it multiplicatively, this seems to be the best available method.

Purely additive and purely multiplicative alternatives were rejected because they do not allow for both the concept of daily weights and a multiplicative relationship of the "trading-day and irregular" component to the trend-cycle. The various alternatives are close approximations to each other, as illustrated by the fact that when two numbers X and Y are in the range of 0.95 to 1.05 (as most monthly calendar adjustment factors and irregulars are), the difference between $Z = X \cdot Y$ and $Z' = X + Y$ is negligible.

Assumption 8 is sometimes violated. The distribution of some irregular components may be skewed. Also, many economic series contain extreme values caused by strikes or unusual events which cannot be considered a part of the "true" irregular distribution. The process of identifying and removing these values may cause a truncation of one or both tails of the distribution, thus leaving a residual with a nonnormal distribution. Since the F-and t-tests are robust against nonnormality, these violations are not considered to be serious.

Standard Errors for Monthly Calendar Adjustment Factors

The regession method provides estimates of 7 daily weights $B_1 \ldots, B_7$ and estimates of their standard errors $\sigma_{B_1}, \ldots, \sigma_{B_7}$.

Monthly calendar adjustment factors are derived from the 7 daily weights as follows (assuming length-of-month variation is left in the seasonal):

31-day months: $$M_{31} = \frac{4\sum_{1}^{7} B_k + B_j + B_{j+1} + B_{j+2}}{31} = \frac{28 + B_j + B_{j+1} + B_{j+2}}{31},$$

where the month begins on day j and $B_{j+7} = B_j$;

30-day months: $$M_{30} = \frac{28 + B_j + B_{j+1}}{30};$$

Leap-year Februaries: $$M_{29} = \frac{28 + B_j}{28.25};$$

Non-leap-year Februaries: $$M_{28} = \frac{28}{28.25} = .991.$$

The standard errors of these factors are then:

$$\sigma^2_{M_{31}} = \left(\frac{1}{31}\right)^2 \left[\sigma^2_{B_j} + \sigma^2_{B_{j+1}} + \sigma^2_{B_{j+2}} + 2\left(\sigma_{B_j B_{j+1}} + \sigma_{B_j B_{j+2}} + \sigma_{B_{j+1} B_{j+2}}\right)\right];$$

$$\sigma_{M_{31}} = \frac{1}{31}\left[\sigma^2_{B_j} + \sigma^2_{B_{j+1}} + \sigma^2_{B_{j+2}} + 2\left(\sigma_{B_j B_{j+1}} + \sigma_{B_j B_{j+2}} + \sigma_{B_{j+1} B_{j+2}}\right)\right]^{\frac{1}{2}};$$

$$\sigma_{M_{30}} = \frac{1}{30}\left[\sigma^2_{B_j} + \sigma^2_{B_{j+1}} + 2\sigma_{B_j B_{j+1}}\right]^{\frac{1}{2}};$$

$$\sigma_{M_{29}} = \frac{1}{28.25}\sigma_{B_j};$$

$$\sigma_{M_{28}} = 0.$$

If length-of-month variation is included in the M's, the denominator of $\sigma_{M_{31}}$, $\sigma_{M_{30}}$ and $\sigma_{M_{29}}$ will be 30.4375.

Appendix B. Derivation of Relationship Between Trading-Day and Irregular Variations

Text table 6 gives levels of $\bar{I}$ above which a set of theoretical "true" trading-day weights with known variance cannot be reliably estimated from observations over a specified time period. These are the levels at which the expected value of the squared deviations between the estimated and true weights are greater than the squared deviations between the true weights and zero. In other words, the critical level is where

$$E\left[\sum_1^7 (b_j - B_j)^2\right] = \sum_1^7 B_j^2,$$

where $E(B_j) = 0$ and $E(b_j) = B_j \quad (j = 1, \ldots, 7)$.

Recalling assumptions 3 to 7 of appendix A,

(Assumption 3′) $Y = XB + E$,

(Assumption 4′) $E(E) = O$,

(Assumption 5′) $V(E) = \sigma^2 I$,

(Assumption 6′) X is fixed,[1]

(Assumption 7′) X has rank = 7,[1]

proceed as follows:

$$b - B = (X'X)^{-1} X'Y - B$$
$$= (X'X)^{-1} X'(XB + E) - B$$
$$= (X'X)^{-1} X'E.$$

$$\text{Then } (b - B)^2 = (b - B)(b - B)'$$
$$= (X'X)^{-1} X'EE'X (X'X)^{-1}$$

$$\text{and } E\left[(b - B)^2\right] = \sigma^2 (X'X)^{-1}.$$

$$\text{Hence, } E\left[(b_j - B_j)^2\right] = \sigma^2 (X'X)^{-1}_{jj} \quad (j = 1, \ldots, 7)$$

$$\text{and } E\left[\sum_1^7 (b_j - B_j)^2\right] = \sigma^2 \sum_1^7 (X'X)^{-1}_{jj}.$$

Let n denote the number of months included in the regression and n_i the number of days in month i $(i = 1, \ldots, n)$.

Make the simplifying assumption:

(Assumption 8′)

All $n_i = 30.4375 \left(\text{average length of month} = \frac{365\ 1/4}{12}\right)$.

Now $Y_i = n_i\left(\frac{I_i}{100} - 1\right)$,

where I_i is the "true" irregular and $E(I_i) = 100$ $(i = 1, \ldots, n)$.

$$\text{Then } \sigma^2 = \sigma_E^2 = \sigma_Y^2 = \frac{n_i^2}{(100)^2}\sigma_I^2$$
$$\doteq \frac{n_i^2}{(100)^2}\left(\frac{(100)^2 \cdot \pi}{4}\right)\bar{I}^2$$
$$= 727.625227\ \bar{I}^2.$$

(For proof that $\sigma_I^2 \doteq \frac{(100)^2 \cdot \pi}{4}\bar{I}^2$, see lemma at end of this appendix.)

Let $K_1 = \sum_1^7 B_j^2$.

$$\text{Now } E\left[\sum_1^7 (b_j - B_j)^2\right] = 727.625227 \sum_1^7 (X'X)^{-1}_{jj}\ \bar{I}^2$$
$$= K_2 \bar{I}^2.$$

[1] Assumptions 6′ and 7′ concerning X are analogous to assumptions 6 and 7 in appendix A concerning $\tilde{X}$.

Then solve for $\bar{I} = \sqrt{\frac{K_1}{K_2}}$

Lemma. Proof of Approximation $\sigma_I^2 \doteq \frac{(100)^2 \cdot \pi}{4}\bar{I}^2$ (due to Harry Rosenblatt of Census):

$$\bar{I} = \frac{1}{n-1}\sum_{t=1}^{n-1} |\delta(I_t)|, \text{ where } \delta(I_t) = \frac{I_{t+1} - I_t}{I_t}.$$

Assume: (Assumption 9′) To a satisfactory approximation, the $\delta(I_t)$ are independent, normal random variables, with zero mean and common variance σ_δ^2.

From assumption 9′ it follows that $\bar{I}$ is a mean deviation with

$$\bar{I} = E|\delta(I)| = \sqrt{\frac{2}{\pi}}\sigma_\delta.$$

From assumption 5′ (the I_t are independent with mean 100 and common variance σ_I^2) and a Taylor expansion approximation to the variance of the ratio represented by $\delta(I_t)$, it follows that

$$\sigma_\delta^2 \doteq \frac{\sigma^2_{I_{t+1}}}{\left[E(I_{t+1})\right]^2} + \frac{\sigma^2_{I_t}}{\left[E(I_t)\right]^2} - \frac{2\,\mathrm{Cov}(I_{t+1}\ I_t)}{E(I_{t+1})E(I_t)}$$
$$\doteq \frac{2\sigma_I^2}{(100)^2}.$$

Hence

$$\sigma_I^2 \doteq \frac{(100)^2}{2}\sigma_\delta^2 = \frac{(100)^2 \cdot \pi}{4}\left[E|\delta(I)|\right]^2,$$

and replacing $E|\delta(I)|$ by its estimate the mean deviation $\bar{I}$ results in the approximation $\sigma_I^2 \doteq \frac{(100)^2 \cdot \pi}{4}\bar{I}^2$.

Appendix C. Importance of Trading-Day Variation in Census Series

The relative contributions of the various components to the month-to-month variation in the unadjusted series, shown in text table 1, are derived from the relation $\overline{O}^2 = \overline{TD}^2 + \overline{E}^2 + \overline{S}^2 + \overline{C}^2 + \overline{I}^2$, where O, TD, . . ., I are the Method II summary measures of average absolute percent changes shown in table C1, (i.e., $\overline{X} = \frac{1}{n-1}\sum_{1}^{n-1}\left|\frac{X_{t+1} - X_t}{X_t}\right|$, where X = O, TD, E, S, C, I) given the assumptions below:

(Assumption 1) O = TD x E x S x C x I; where O, TD, . . ., I designate the original series and its components;

(Assumption 2) $E\left[\delta(X)\right]$ = (zero), where $\delta(X) = \left|\frac{X_{t+1} - X_t}{X_t}\right|$ and X = O, TD, E, S, C, I;

(Assumption 3) the $\delta(X)$ have the same distribution, where X = O, TD, E, S, C, I;

(Assumption 4) TD, E, S, C, and I are independent.

Assumption 2 is violated for O and C when the series contains a trend. However, the errors tend to be offsetting. Since the assumptions do not hold exactly, the following relation is used in order to force the results to add to exactly 100 percent. $\overline{O}^2 \doteq \overline{O}'^2 = \overline{TD}^2 + \overline{E}^2 + \overline{S}^2 + \overline{C}^2 + \overline{I}^2$. These formulations were developed by Bongard at the OECD (1) and Rosenblatt at Census.

Table C2 shows the daily weights from which the monthly trading-day components are derived.

Table C1.--SUMMARY MEASURES OF CENSUS SERIES

(Average month-to-month percent change, without regard to sign)

Census series	Un-adjusted series $\overline{O}$	Trading-day component $\overline{TD}$	Holiday component $\overline{E}$	Seasonal component[2] $\overline{S}$	Trend-cycle component $\overline{C}$	Irregular component $\overline{I}$
Sales of retail business, 1953-63	7.50	1.96	0.32	6.75	0.43	0.65
Sales of wholesale business, 1960-63	5.56	3.52	....	3.55	0.38	0.87
Manufacturers' shipments, 1953-62[1]	5.27	2.24	....	4.24	0.79	1.18
Manufacturers' new orders, 1953-62[1]	5.10	2.24	....	4.04	1.00	1.93
U.S. exports, 1953-63	6.18	2.61	....	4.85	1.02	2.43
U.S. imports, 1953-63	7.00	5.20	....	4.53	0.91	2.32
New building permits, private housing units, 1954-62[1]	11.74	5.27	....	9.52	1.56	3.20

[1]Summary measures obtained from seasonal adjustment of aggregate series rather than from sum of seasonally adjusted components.
[2]Length-of-month variation is included in the seasonal component.

Table C2.--DAILY WEIGHTS FOR CENSUS TRADING-DAY ADJUSTMENTS

Census series	Total	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.
Sales of retail business, 1953-64:								
Nondurable goods[1]	7.00	0.87	0.92	0.93	0.98	1.43	1.38	0.49
Durable goods[1]	7.00	1.27	0.97	1.20	1.01	1.55	0.67	0.33
Sales of wholesale business, 1960-64[1]	7.00	0.96	1.26	1.48	1.16	1.21	0.53	0.40
Manufacturers' shipments, 1953-64	7.00	1.17	1.17	1.17	1.17	1.17	0.58	0.58
Manufacturers' new orders, 1953-64	7.00	1.17	1.17	1.17	1.17	1.17	0.58	0.58
U.S. exports:								
1953-60	7.00	1.00	1.00	1.05	1.10	1.85	0.75	0.25
1961-64	7.00	0.75	1.10	1.00	1.05	1.70	0.90	0.50
U.S. imports, 1953-64	7.00	1.61	1.47	1.33	1.33	1.26	0.00	0.00
New building permits, private housing units, 1954-64	7.00	1.40	1.40	1.40	1.40	1.40	0.00	0.00

[1]Daily weights are weighted averages of weights used for individual kinds of business.