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ALTERNATIVE SETS OF WEIGHTS FOR PROPOSED X-11 SEASONAL FACTOR CURVE MOVING AVERAGES

The following tables contain the end weights computed by an "optimum" technique for the 4 seasonal factor moving averages proposed for inclusion in X-11. These sets of weights will provide the minimum amount of revision between seasonal factors computed using these weights and factors computed using the corresponding central weights under the assumption that the seasonal pattern in the future contains a linear trend. The weights applied to the central terms are denoted by W_i, while a_i are the weights for the next-to central term, b_i are the weights the term 2 years from the central terms, etc. The central terms are denoted by N, while the (N + 1) term has one end value missing, etc.

The technique used to compute these end weights is described in an earlier draft. The end weights are a function of:

(a) the central weights W_i;

(b) $\bar{I}'/\bar{S}'$, the moving seasonality ratio (MSR);

(c) the level of the seasonal for the month under consideration.

Algebraically, the end weights are a function of the W_i and

$$D = \frac{\Delta_S^2}{\sigma_I^2} = \frac{4}{\pi}\left(\frac{1}{100\ H}\right)^2\left(\frac{1}{MSR}\right)^2$$

where σ_I^2 is the variance of the irregular, Δ_S^2 is the square of the average month-to-month change in the seasonal, $H = \sum_1^n \frac{1}{S_t}$ is the harmonic mean of the seasonal factors for the particular month, n = number of years of data available for the particular month.

These end weights tell us nothing about the properties of the moving averages used for the central terms. They merely guarantee the smallest revisions for a given set of central weights. Their validity is based on the following assumptions:

(1) the 7-term moving average of the S – I ratios provides an accurate preliminary estimate of S;

(2) the ratios of the S – I ratios to the 7-term moving average are an accurate preliminary estimate of I;

(3) the I_i (historical and future) are independent with equal variance σI^2 ;

(4) $(SI)_i = S_i + I_i$; $E(I_i)=0$, $\sigma_{S_i I_i} = 0$.

(5) the seasonal pattern in the current and future periods contains a linear trend which has change Δs equal to the average absolute year-to-year change in S during the historical period.

The weights given in the following table are based on the assumption that $H = \frac{1}{100}$ (i.e., the level of S is approximately 100). As the level of S increases, more weight is shifted to the end terms in the average. (Also, as the degree of moving seasonal increases [the MSR decreases], more weight is shifted to the end terms.) Following is an example of the effect of the level of the seasonal on the end-weight pattern.

3 X 3 Moving Average — End (N + 2) Term — MSR = 2.00

$\frac{1}{H}$	C_1	C_2	C_3
80	.109	.333	.556
90	.091	.333	.576
95	.080	.333	.586
100	.071	.333	.595
105	.062	.333	.605
110	.053	.333	.613
120	.036	.333	.633

End Weights for 3 - Term Moving Average

Central (N) Term Weights

W_1	W_2	W_3
.333	.333	.333

(N + 1) Term Weights

MSR	A_1	A_2
0.25	.045	.958
0.50	.141	.859
0.75	.235	.766
1.00	.305	.694
1.25	.355	.645
1.50	.390	.608
Stable	.500	.500
X - 10	.333	.666
X - 11*	.500	.500

*- Preliminary